Insider Trading and Voluntary Disclosure

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Abstract

I investigate the incentives of insiders from publicly traded companies to voluntarily disclose their private information. This is done in a multi-asset Kyle-type trading framework where disclosure serves to attract discretionary liquidity trades. Equilibria where some insiders completely disclose their private information only exist when at least two assets are not traded by non-discretionary liquidity traders, referred to as noise traders. When at most one asset has no noise trading, then all insiders retain some information and trade upon it. The amount of information retained by insiders in this case increases with noise trading volume and equilibria where insiders do not reveal any information at all are more likely to exist as the number of assets increases.

Keywords: Insider trading; Liquidity trading; Disclosure

JEL classification numbers: D82; G12

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1 Introduction

Publicly traded companies disclose significant amounts of information on a voluntary basis. Public firms spend resources not only in investor relations but also in trying to attract the attention of analysts who in turn produce information through research reports. There is abundant evidence, as summarized in Brennan and Tamarowski (2000), that analysts prefer following companies that provide them with information on a timely basis and that increased analyst coverage improves liquidity. On the other hand, the abnormal profits associated with corporate insiders’ trades (Seyhun, 1986 and 1992 and Meulbroek, 1992) suggest that insiders’ information is not entirely revealed to the markets. This raises the following questions, which I attempt to answer in this paper: To which extent can investors rely on the information provided by public firms? Are there firms for which insiders are expected to make more precise announcements than others?

To this end, I develop a static trading model in the spirit of Kyle (1985) with many assets, one insider per asset, many noise traders, many market makers, one discretionary liquidity trader. All agents in the model are risk neutral. Each insider becomes informed of the liquidation value of her firm before all other market participants and have the possibility to trade upon this private information. The discretionary liquidity trader (liquidity trader hereafter) can allocate his trades across the different assets in order to minimize his expected trading loss. Before trade takes place, a public signal is sent for each asset, consisting of a noisy transformation of its insider’s private information. The insider’s disclosure decision corresponds to choosing the precision of her asset’s public signal, represented by the inverse of the variance of the noise in the signal. A stock may or may not be traded by noise traders, who also trade for liquidity reasons but who cannot trade in stocks other than the one they are assigned to. Prices are determined by competitive market
makers who only observe the aggregate order flow in their respective stock.

In the present model, the disclosure of private information by an insider reduces the price sensitivity of her stock to the size of order flows. This in turn reduces the liquidity trader's cost of trading in that stock and thus more of the latter's trades will be allocated to it, allowing the insider to trade more aggressively. Hence an insider will choose to disclose more information if this translates into greater trading profits, as is the case in Chowdhry and Nanda (1991) and Huddart, Hughes and Brunnermeier (1999). The former article analyzes the trading behavior of an insider whose stock is listed on more than one exchange and the latter investigates insiders' listing decisions, in a multi-asset setting, between exchanges with different disclosure requirements. The framework presented here differs from these two papers in that insiders can freely determine their disclosure policy instead of complying to exchange regulations. It differs from Chowdry and Nanda by using a multi-asset environment and it differs from Huddart et al. by allowing for the presence of noise traders.

I find that only full-disclosure equilibria, defined as equilibria where some insiders completely reveal their private information, exist when at least two assets do not have any noise trading in them. On the other hand, the presence of noise trading in all assets, or all assets but one, only induces equilibria where all insiders retain some information. If the volume of noise trading in each asset is sufficiently large, then insiders do not disclose any information at all, and this no-disclosure equilibrium is more likely as the number of assets increases. Also, it is shown here that the insider of an asset without noise trading reveals more precise information than all other insiders when noise traders are present in all assets but one.

These results contrast with those of Huddart et al. (1999), where insiders always choose to list their company on the stock exchange with the highest disclosure requirement. This difference
is due to the presence of noise traders in the present model, this type of traders being absent
from Huddart et al.’s framework. Chowdhry and Nanda (1991), on the other hand, allow for both
discretionary and non-discretionary liquidity trading, and show that informed and uninformed
trades concentrate in markets with more stringent disclosure policies as the volume of discretionary
liquidity trading becomes arbitrarily large. Chowdhry and Nanda, however, consider a single asset
trading at different locations whereas I propose a multi-asset model. A multi-asset setting enlarges
the analysis as disclosure decisions depend on the number of assets.

The results found herein have two major implications. Firstly, they suggest that executives from
thinly traded firms, represented here by firms without noise trading in their stock, have incentives
to give more precise information than executives from thickly traded firms. Hence managers of
portfolios consisting of small companies with low trading volumes may put more weight on what
the executives of these companies have to say than managers of portfolios of large companies.
If companies were perfectly transparent, there would be no need of analyst reports. As shown in
Bhushan (1989), analysts prefer following large, high trading volume, firms. However, the difference
in analyst coverage cannot be entirely attributed to the fact that large firms have more shareholders
and thus a greater demand for analyst reports since small, low volume, firms have much greater
expected returns and thus investors should be willing the pay in consequences. Hence differences
in the precision of disclosure statements may explain why the demand for analyst reports is smaller
for low volume firms.

Secondly, it is shown that insiders reveal less precise information as the number of assets increase.
That is, the more diversified the discretionary trader can be, the less he suffers from information
asymmetries and thus the less sensitive he is to insiders’ disclosure. Hence insiders do not need
to reveal a lot of information in order to have their profit-maximizing share of liquidity trades.
Regarding this point, the spectacular increase in the number of mutual funds over the past decades may have had a perverse effect on the precision of firms’ disclosure. As shown in Bhattacharya and Daouk (2002), countries have been progressively adopting insider trading laws in the past decades, and this may be due to the increase in the number of securities available to invest in, which reduces insiders’ incentives to disclose information and thus increases the occurrence of insider trading.

The paper is structured as follows: The next section surveys some of the literature in this area of research, Section 3 describes the model, Section 4 analyzes the players’ trading behaviors, Section 5 characterizes disclosure strategies and Section 6 concludes.

2 Related Literature

The present paper follows a stream of literature initiated by Kyle (1985), who studies the profit-maximizing use of private information by a monopolist insider in a setting where information asymmetry creates endogenous trading costs. These costs arise from the market maker’s pricing function, whose sensitivity to order flows increases with the quality of the insider’s private information.

Admati and Pfleiderer (1988), Foster and Viswanathan (1990) and Bhushan (1991) have shown that traders motivated by liquidity needs minimize their trading costs by directing their trades in assets with less information asymmetry. Admati et al. and Foster et al. also show that informed traders tend to trade at the same time as liquidity traders since the presence of the latter allows the former to trade more aggressively, which in turn increases their trading profits. Chowdhry and Nanda (1991) and Huddart, Hughes and Brunnermeier (1999) have shown that insiders may even prefer trading in markets with more stringent disclosure requirements in order to benefit from the
presence of discretionary liquidity traders.

Hong and Huang (2001) suggest that an insider may be willing to create market depth in her stock through disclosure because she may eventually be forced to sell some of her holdings for liquidity reasons in the future, and the cost of doing so decreases as the precision of her disclosure increases. Hong and Huang also relate the precision of the insider’s statement to her share of ownership of the firm.

Verrechia (1983) proposes a model where the seller of an asset has to choose between revealing or concealing information about the asset. In this framework, revealing information is costly but concealing it can be perceived as a bad signal by investors who then bid low prices for the asset. Narayanan (2000) makes endogenous the disclosure costs in Verrechia’s model by allowing the seller of the asset to trade upon his private information. In these two papers, part of the insider’s compensation is positively linked to the asset price, feature without which the insider would never reveal any information. In the present model, an insider’s disclosure decision is not meant to boost the stock price, its purpose is to attract liquidity trades.

Bushman and Indjejikian (1995) and Shin and Singh (1999) show that a trader with superior information may choose to reveal some of it in order to erode the competitiveness of other traders with information superior to that of the market maker. The present model assumes a single insider per asset and adding more insiders would definitely affect insiders’ disclosure decisions. Nevertheless, this would leave the results qualitatively unaffected in the sense that equilibria with and without full disclosure would exist under the same conditions.
3 The Model

Consider a one-period trading model with $M$ firms denoted $m = 1, \ldots, M$. The liquidation value of firm $m$'s stock is given by $V_m \sim N(0, \sigma_v^2)$ (Throughout the paper, uppercases are used to denote random variables and lowercases denote their realization) and asset values are not correlated. There is one insider per asset, one discretionary liquidity trader, many competitive market makers and many noise traders. Trading is as in Kyle (1985) and there is no restriction on short sales.

Each insider knows exactly the value of her asset before trade begins and her only goal is to profit from her private information. Insider $m$ only trades in asset $m$, the number of shares she trades is denoted $x_m$ and her ex-post payoff is given by $\pi_m = (v_m - p_m)x_m$, $p_m$ representing the price of asset $m$. Each insider must also choose a disclosure policy which determines the precision of a public signal to be sent before trade begins. This signal is given by

$$S_m = V_m + \epsilon_m,$$

where $\epsilon_m \sim N(0, \sigma_{\epsilon_m}^2)$. Insider $m$’s disclosure decision consists of choosing $\sigma_{\epsilon_m}^2$, which is done before learning $V_m$. This way of modeling disclosure, which is very common in this literature (See, for example, Admati and Pfleiderer (1986), Shin and Singh (1999), and Huddart et al. (1999)), can be seen as a long-term commitment comparable to firms’ relationships with financial analysts. As is well documented (see Brennan and Tamarowski (2001)), analysts prefer covering firms that provide them with accurate, timely, information and these are rarely given a second chance.

No generality is lost under this setting, as $\sigma_{\epsilon_m}^2 = 0$ corresponds to full disclosure and $\sigma_{\epsilon_m}^2 = \infty$ corresponds to no disclosure at all. Note that insiders are assumed to tell the truth on average, they cannot intentionally manipulate the market. All $\epsilon_m$’s are independent variables. The vectors $S = (S_1, \ldots, S_M)$ and $\Sigma_{\epsilon} = (\sigma_{\epsilon_1}^2, \ldots, \sigma_{\epsilon_M}^2)$ are common knowledge.
There is one discretionary liquidity trader whose overall market order is exogenously given by $U \sim N(0, \sigma_u^2)$. The motivations behind $U$ are not modeled here; these could be tax concerns or portfolio rebalancing reasons. The liquidity trader can allocate his trades across the different assets in order to maximize his trading profits. Due to the presence of insiders, this problem translates into minimizing trading losses. The fraction of $U$ allocated to asset $m$ is denoted $g_m$, with $g_m \in [0, 1]$ and $\sum m g_m = 1$. Let $g = (g_1, \ldots, g_M)$, which will be referred to as the liquidity allocation.

Noise trading occurs randomly in each asset, where $Z_m \sim N(0, \sigma_z^2)$ denotes the net purchases of firm $m$’s shares by noise traders. The absence of noise trading in asset $m$ will be characterized by $\sigma_z^2 = 0$. The variables $Z_1, \ldots, Z_M$ are all independently distributed and $\Sigma_z = (\sigma_z^2, \ldots, \sigma_z^2)$ is common knowledge.

Trading orders are simultaneously submitted to market makers who only observe the aggregate order flow in their asset, given by $y_m = x_m + g_m u + z_m$, $m = 1, \ldots, M$. Market makers set efficient prices given their information, i.e. $p_m = E[V_m|s_m, y_m]$ for all $m$. This implies that market makers do not expect to profit from their market-clearing trades, which can be seen as the result of a Bertrand competition between them. Note also that a market maker does not possess any prior information on the asset he trades, unlike in Jain and Mirman (1999). The timing of events is depicted in Figure 1. Table 1 shows the information of each agent at each point in time.

4 Trading

The goal of this paper is to characterize insiders’ disclosure policies $\Sigma_\epsilon$ under different noise trading structures $\Sigma_z$. To do so, I first need to know the trading outcomes that may occur for a given disclosure policy profile since this gives us insiders’ expected payoffs as functions of $\Sigma_\epsilon$. Once this
Table 1: Information structure throughout the game.

<table>
<thead>
<tr>
<th>time</th>
<th>Insider m</th>
<th>Liquidity Trader</th>
<th>Market Maker m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>(\Sigma_z)</td>
<td>(u, \Sigma_z)</td>
<td>(\Sigma_z)</td>
</tr>
<tr>
<td>1 - 2</td>
<td>(v_m, s, \Sigma_z, \Sigma_\epsilon)</td>
<td>(u, s, \Sigma_z, \Sigma_\epsilon)</td>
<td>(s, \Sigma_z, \Sigma_\epsilon)</td>
</tr>
<tr>
<td>3</td>
<td>(v_m, s, \Sigma_z, \Sigma_\epsilon)</td>
<td>(u, s, \Sigma_z, \Sigma_\epsilon)</td>
<td>(s, \Sigma_z, \Sigma_\epsilon, y_m)</td>
</tr>
<tr>
<td>4</td>
<td>everything</td>
<td>everything</td>
<td>everything</td>
</tr>
</tbody>
</table>

is done, I will be able find insiders’ strategic disclosure decisions. Hence, throughout the rest of this section, \(\Sigma_\epsilon\) is taken as given.

Attention is restricted to equilibria where \(p_m\) is a linear function of \(y_m\). In such an equilibrium, prices are, for all \(m\) (derivations are in appendix),

\[
p_m(s_m, y_m) = E[V_m|s_m] + \lambda_m y_m,
\]

where \(E[V_m|s_m] = \sigma^2_{m|s} \sigma^2_v\) and \(\lambda_m = \frac{\sqrt{\text{var}(V_m|s_m)}}{2\sqrt{\sigma^2_v + \sigma^2_{m|s}}}\), \(\overline{g}_m\) being Market Maker \(m\)'s (correct) anticipation of \(g_m\). To allege notation, \(\sigma^2_{m|s}\) will be used hereafter to denote \(\text{var}(V_m|s_m)\).

Note that \(\sigma^2_{m|s} = \frac{\sigma^2_v \sigma^2_{m|s}}{\sigma^2_v + \sigma^2_{m|s}}\), which is strictly increasing in \(\sigma_{m|s}\), with \(\lim_{\sigma_{m|s} \to 0} \sigma_{m|s} = 0\) and \(\lim_{\sigma_{m|s} \to \infty} \sigma_{m|s} = \sigma_v\). It is therefore simpler to use \(\sigma_{m|s}\) as the control variable for disclosure, which is what I will be doing from now on. Let then \(\Sigma_s = (\sigma_{1|s}, \ldots, \sigma_{M|s})\), that will be referred to as the vector of insiders’ information advantages, and let \(\Sigma_{s,-m}\) denote \(\Sigma_s\) without its \(m\)th entry. For all \(m\), \(\sigma_{m|s}\) measures the difference between the variance of \(V_m\) as perceived by traders other than Insider \(m\) and the variance of \(V_m\) from Insider \(m\)'s viewpoint.

As in Kyle (1985), \(\lambda_m\) captures relevant information in \(y_m\) that is not contained in \(s_m\). If, for
example, Insider $m$ decides to retain some information by choosing $\sigma_{\epsilon m} > 0$ and thus $\sigma_{m|s} > 0$, then $\lambda_m$ is positive and is increasing in $\sigma_{m|S}$. In this case, an aggregate buy order ($y_m > 0$) means that Insider $m$ may also be buying and thus stock $m$ is likely to be underpriced at $E[V_m|s_m]$ (a similar reasoning applies to aggregate sell orders). If Insider $m$ is the only potential trader in stock $m$, i.e. if $\sigma_{u}^2 g_m^2 + \sigma_{z}^2 = 0$, then Market Maker $m$ knows that any trade in stock $m$ comes from the insider and thus his supply curve has an infinite slope to prevent any loss from these trades. The presence of uninformed trading in stock $m$ ($\sigma_{u}^2 g_m^2 + \sigma_{z}^2 > 0$), on the other hand, reduces the likelihood of inside trades and thus the slope the market maker’s supply curve. If Insider $m$ has given up all of her information advantage through her disclosure policy, i.e. if $\sigma_{m|s} = 0$, then $y_m$ does not contain any information at all and $\lambda_m = 0$.

In this game, traders take into account the impact of their trades on stock prices but take the functional form of the pricing functions, given by $E[V_m|s_m]$ and $\lambda_m$, $m = 1, \ldots, M$, as given. In the case of the discretionary liquidity trader, only $\lambda = (\lambda_1, \ldots, \lambda_M)$ matters in his allocation decision since $E[Z_m|u, s_m] = E[X_m|u, s_m] = 0$ and thus, for all $m$,$$E \left[ (V_m - p_m(S_m,Y_m)) g_m U \bigg| U = u, S_m = s_m \right] = -\lambda_m g_m^2 u^2. \quad (2)$$Hence the liquidity trader’s problem consists of minimizing $\sum_{m=1}^{M} \lambda_m g_m^2 u^2$ such that $\sum_m g_m = 1$ and $g_m \geq 0$ for all $m$. Note that the solution to this problem depends on $\lambda$ which in turn depends $\Sigma_s$ as previously shown. The solution to the liquidity trader’s problem will then be denoted $g(\Sigma_s) = (g_1(\Sigma_s), \ldots, g_M(\Sigma_s))$.

If $\sigma_{m|s} = 0$ for some $m$, then the liquidity trader only trades in assets whose insiders have no information advantage since this assures him of a zero trading loss. In this case, his liquidity allocation can be any allocation such that all the trades are made in assets with $\sigma_{m|s} = 0$. 
If, on the other hand, $\sigma_m|s > 0$ for all $m$, then

$$g_m(\Sigma_s) = \frac{1}{\lambda_m \sum_{k=1}^{M} \frac{1}{\lambda_k}}$$

for all $m$. That is, $g_m(\Sigma_s) > 0$ for all $m$ such that $\lambda_m < \infty$ and $\lambda_m g_m(\Sigma_s) = \lambda_{m'} g_{m'}(\Sigma_s)$ for all $m, m'$ such that $\lambda_m, \lambda_{m'} < \infty$.

If the market maker correctly anticipates the liquidity trader’s allocation, then $\lambda_m = \frac{\sigma_m|s}{2\sqrt{\sigma_u^2 g_m(\Sigma_s)^2 + \sigma_{z_m}^2}}$, and $\lambda_m g_m = \lambda_{m'} g_{m'}$ can be rewritten as

$$\frac{\sigma^2_{m|s}}{\sigma_u^2 + \sigma_{z_m}^2 / g_m(\Sigma_s)^2} = \frac{\sigma^2_{m'|s}}{\sigma_u^2 + \sigma_{z_{m'}}^2 / g_{m'}(\Sigma_s)^2}$$

for all $m, m'$ such that $\lambda_m, \lambda_{m'} < \infty$.

Since $\lambda_m < \infty$ whenever $\sigma_u^2 g_m + \sigma_{z_m}^2 > 0$, and since we cannot have $g_m(\Sigma_s) = 0$ for all $m$, equation (4) implies that $g_m(\Sigma_s) > 0$ for all $m$ with $\sigma^2_{z_m} > 0$. That is, the presence of noise traders in a stock implies that a positive fraction the discretionary trades are allocated to that stock.

Let $\sigma_s = \min_m \sigma_{m|s}$, let $M = \{ m : \sigma_{m|s} = \sigma_s \}$ and let $\gamma(\Sigma_s) = \frac{\sigma^2_{m|s}}{\sigma_u^2 + \sigma_{z_m}^2 / g_m(\Sigma_s)^2}$ for all $m$ with $\lambda_m < \infty$. In words, $M$ is the set of assets with the least information asymmetry and $\frac{1}{2} \sqrt{\gamma(\Sigma_s)}$ is the discretionary trader’s expected loss given the information advantage profile $\Sigma_s$. As shown in appendix, an asset without noise trading (no-noise asset hereafter) will be traded by the discretionary trader only if it belongs to $M$ and only if there is no asset in $M$ with positive noise trading (asset with noise hereafter). That is, the discretionary liquidity trader never trades in no-noise assets if there exists an asset with noise and the same information asymmetry. Moreover, the liquidity allocation $g(\Sigma_s)$ is unique whenever $\sigma_{m|s} > 0$ for all $m$ and $M$ either contains noise assets or contains only one no-noise asset. If $M$ contains two or more no-noise assets, then the the overall fraction of liquidity trades allocated to assets in $M$ and the fraction allocated to assets not
in $\mathcal{M}$ is unique but there is an infinity of ways trades can be allocated across assets in $\mathcal{M}$. These results are summarized in the following lemma, which details are in appendix.

**Lemma 1** Let $\mathcal{M}$, $\sigma_s$ and $\gamma(\Sigma_s)$ be as described above and let $g(\Sigma_s) = (g_1(\Sigma_s), \ldots, g_M(\Sigma_s))$ denote the allocation that solves the liquidity trader’s problem given an information advantage profile $\Sigma_s$. Then $g(\Sigma_s)$ is as follow:

1. If $\sigma_{m|s} = 0$ for some $m$, then $g_m(\Sigma_s)$ can be any allocation such that $\sum_m g_m = 1$ and

   \[ g_m \geq 0 \text{ if } \sigma_{m|s} = 0, \]
   \[ g_m = 0 \text{ if } \sigma_{m|s} > 0. \]

2. If $\sigma_{m|s} > 0$ for all $m$, then

   \[ g_m(\Sigma_s) = \frac{\sigma_{zm}}{\sqrt{\sigma_{m|s}^2 / \gamma(\Sigma_s) - \sigma_u^2}} \]

   for all $m$ such that $\sigma_{zm} > 0$. If an asset $m'$ is such that $\sigma_{zm'} = 0$, then $g_m'(\Sigma_s) \geq 0$ only if $m' \in \mathcal{M}$ and $\sigma_{zm} = 0$ for all $m \in \mathcal{M}$.

Note that there won’t be any liquidity trades allocated to assets in $\mathcal{M}$ when $\sigma_{zm} = 0$ for all $m \in \mathcal{M}$ if $\sigma_s$ is not sufficiently small. For example, suppose assets $m$ and $m'$ are such that $\sigma_{zm} = 0$ and $\sigma_{zm'} > 0$. Then, if $\sigma_{m|s}^2 \geq \frac{\sigma_{m'|s}^2}{1+\sigma_{zm'}^2 / \sigma_u^2}$, we must have $g_m = 0$ otherwise equation (4) only holds if $g_{m'} > 1$, which is not possible. It is nevertheless always possible, when $\sigma_{m|s} > 0$ for all $m$, for the insider of a no-noise asset to choose a partial disclosure policy such that the fraction of liquidity trades in her asset is positive, where partial means that the insider retains some information.
5 Disclosure Policies

When choosing her disclosure policy, an insider does not know the liquidation value of their asset, but the structure of noise trading across asset, $\Sigma_z$, is known to all. At this stage of the game, a strategy profile $\Sigma_s$ yields an expected payoff of

$$E \left[ \pi_m(V_m, S_m, Y_m) \mid \Sigma_s \right] = \frac{\sigma_{m|s}}{2} \sqrt{\sigma_u g_m(\Sigma_s)^2 + \sigma_m^2}$$

(5)

to Insider $m$, and this for all $m$.

Disclosure decisions are made simultaneously by insiders. As I have shown earlier, choosing $\epsilon_m$ is equivalent to choosing $\sigma_{m|s}$ and thus the latter will be used as Insider $m$’s disclosure policy. Let then $A_m = [0, \sigma_v]$ denote Insider $m$’s set of actions, let $\Theta_m$ denote the set of probability distributions over all Borel subsets of $A_m$ and let $\theta_m$ represent an element of $\Theta_m$. Define $A$ and $\Theta$ as the Cartesian products $A = A_1 \times \ldots \times A_M$ and $\Theta = \Theta_1 \times \ldots \times \Theta_M$. Given a strategy profile $\theta = (\theta_1, \ldots, \theta_M) \in \Theta$, an insider’s expected payoff is given by

$$u_m(\theta) = \int_{A_1} \int_{A_2} \ldots \int_{A_M} \frac{\sigma_{m|s}}{2} \sqrt{\sigma_u g_m(\Sigma_s)^2 + \sigma_m^2} d\theta_1 d\theta_2 \ldots d\theta_M$$

A Nash equilibrium is therefore a strategy profile $\theta^* \in \Theta$ such that

$$u_m(\theta^*) \geq u_m(\theta_m, \theta_{-m})$$

for all $\theta_m \in \Theta_m$ and for all $m$, where $\theta_{-m}$ denotes the vector $\theta$ without its $m$th entry. The payoff functions being continuous and strategy spaces being nonempty compact subsets of a metric space, this game always has a mixed strategy equilibrium.

First note that Insider $m$’s expected payoff is at least $\frac{1}{2} \sigma_v \sigma_m$, when she does not disclose any information, i.e. when $\sigma_{m|s} = \sigma_v$. Hence the insider of an asset with noise trading will never
completely disclose her private information since this would provide her with a zero expected payoff, while not disclosing any information gives her a positive expected payoff. This is not the case, however, for insiders of no-noise assets whose expected payoff is positive only if they disclose at least as much information as all other insiders.

Suppose, for example, that assets \(m'\) and \(m''\) are such that \(\sigma_{zm'} = \sigma_{zm''} = 0\) and consider a disclosure policy profile where \(\sigma_{m'|s} = \sigma_{m''|s} = 0\). Then neither Insider \(m'\) nor Insider \(m''\) has an incentive to deviate since their expected payoff is zero regardless of their play. Expected payoffs are also zero for all insiders of no-noise assets regardless of their disclosure policy. In the case of assets with noise trading, their insider’s expected payoff is \(\frac{1}{2}\sigma_{m|s}\sigma_{zm}\) since none of the liquidity trades are allocated to these assets. Hence these insiders’ best reply consists of not disclosing any information at all. Equilibria where at least two insiders completely disclose their private information will be referred to as full-disclosure equilibria. As shown in appendix, only full-disclosure equilibria exist when at least two assets have no noise trading in them.

If, on the other hand, there is at most one no-noise asset, then a full-disclosure equilibrium does not exist. As shown above, insiders of assets with positive noise trading always retain some information and thus there always exists an information advantage for which the no-noise asset is being allocated some of the liquidity trades. Hence full disclosure is never optimal for the insider of the no-noise asset since this would provide her with a zero expected payoff.

If there is sufficient noise trading volume in each asset, then the no-disclosure outcome, i.e. \(\sigma_{m|s} = \sigma_v\) for all \(m\), is an equilibrium. Typically, whether this outcome is an equilibrium depends on the number of assets and the smallest volume of noise trading. That is, suppose asset \(m'\) is such that \(\sigma_{zm'} \leq \sigma_{zm}\) for all \(m\). If \(\sigma_{m'|s} = \sigma_v\) is Insider \(m'\)'s best reply to \(\sigma_{k|s} = \sigma_v\) for all \(k \neq m'\), then \(\sigma_v\) is also a best reply for all other insiders.
The no-disclosure outcome, however, is never an equilibrium when one asset has no noise trading. As we have seen in Lemma 1, the fraction of liquidity trades allocated to a no-noise asset is positive only if its insider discloses more information than all insiders in assets with noise trading. Hence if 
$$\sigma_{z_{m'}} = 0,$$
say, and 
$$\sigma_{z_m} > 0$$
and 
$$\sigma_{m|s} = \sigma_v$$
for all 
$$m \neq m',$$
then Insider 
$$m'$$'s expected payoff with 
$$\sigma_{m'|s} = \sigma_v$$
is zero whereas 
$$\sigma_{m'|s}$$ sufficiently small would guarantee a positive liquidity allocation to asset 
$$m'$$ and thus a positive expected payoff to Insider 
$$m'$$. The following proposition summarizes the above results, which are proved in appendix.

**Proposition 1** Disclosure policies in the trading game described above are as follows:

1. If there is no noise trading in at least two assets, i.e. if 
$$\sigma_{z_m} = 0$$
for at least two assets, then the insiders of at least two of these assets fully disclose their private information. Insiders in assets with noise trading do not disclose any information and the remaining no-noise asset insiders may choose any disclosure level. In this equilibrium, expected payoffs are zero for insiders of no-noise assets and 
$$\frac{1}{2}\sigma_v \sigma_{z_m}$$
for insiders of assets with noise trading.

2. If there is at most one asset without noise trading, then full disclosure is not an equilibrium.

If noise traders are present in all assets, then no disclosure by all insiders, i.e. 
$$\sigma_{m|s} = \sigma_v$$
for all 
$$m,$$
is an equilibrium if 
$$\sigma_{z_m}$$
is sufficiently large for all 
$$m.$$ If there is exactly one no-noise asset, then the insider of this asset always discloses some information in equilibrium.

One implication of these results is that the information coming from less frequently traded stocks, represented here by stocks without noise trading, is more reliable than the information coming from firms with high trading volume. In a survey conducted in 2003, the Association for Investment Management and Research (AIMR) found that portfolio/fund managers and security
analysts gave an average grade of 3.4 out of 5 to the overall quality of information public companies choose to disclose. It would now be interesting to see how this grade is related to company size and trading volume.

In what follows, we will make a simplifying assumption that will help us find analytical solutions to insiders’ disclosure problem. That is, let us assume from now on that

\[ \sigma_{zm} = \sigma_z > 0 \]  \hspace{1cm} (A)

for all \( m \). The case \( \sigma_z = 0 \) will not be discussed since it yields full disclosure equilibria just as in the case with heterogeneous noise trading and two or more assets with \( \sigma_{zm} = 0 \).

Let us first consider pure strategy equilibria. As shown in appendix, the only possible symmetric pure strategy equilibrium when \( M \geq 3 \) is \( \sigma_{m|s} = \sigma_v \) for all \( m \) (the no-disclosure equilibrium hereafter), and this equilibrium exists only if \( \sigma_z^2 \) is sufficiently larger than \( \sigma_u^2(1/M^2 - 1/M^3) \). When \( M \geq 3 \) and \( \sigma_z^2 = \sigma_u^2(1/M^2 - 1/M^3) \), then \( \sigma_{m|s} = \sigma_v \) is a minimum of \( u_m(\sigma_{m|s}, \sigma_v) \), where \( \sigma_v \) is the vector \( \theta_{-m} \) with all entries equal to \( \sigma_v \). When \( M \geq 3 \) and \( \sigma_z^2 \) is slightly greater than \( \sigma_u^2(1/M^2 - 1/M^3) \), then \( u_m(\sigma_{m|s}, \sigma_v) \) is as in Figure 2 Case II, in which case \( \sigma_{m|s} = \sigma_v \) is not a best reply to \( \sigma_v \) either. If, on the other hand, \( \sigma_z^2 \) is sufficiently larger than \( \sigma_u^2(1/M^2 - 1/M^3) \), then \( \sigma_{m|s} = \sigma_v \) is a best reply to \( \sigma_v \) and thus the no-disclosure outcome is then an equilibrium.

The story is different when \( M = 2 \). In this case, the no-disclosure equilibrium exists if and only if \( \sigma_z^2 \geq \sigma_u^2(1/M^2 - 1/M^3) = \sigma_u^2/8 \). When \( M = 2 \) and \( \sigma_z^2 = \sigma_u^2/8 \), other symmetric pure strategy equilibria exist as well, as long as the information asymmetry in each asset is not too small.

If \( \sigma_z^2 < \sigma_u^2(1/M^2 - 1/M^3) \), in which case \( u_m(\sigma_{m|s}, \sigma_v) \) is as in Figure 2, Case I, or if \( \sigma_z^2 \geq \sigma_u^2(1/M^2 - 1/M^3) \) and the no-disclosure outcome is not an equilibrium, then a symmetric equilibrium in pure strategies does not exist. In this case, there exists a mixed strategy equilibrium where all
insiders randomize between $\sigma_v$ and some $\hat{\sigma}_s \in (0, \sigma_v)$, as shown in appendix\(^1\). This leads to the next proposition.

**Proposition 2** Consider the trading game described above where $\sigma_{zm} = \sigma_z > 0$ for all $m$. Then equilibria are as follows.

(i) When $M = 2$, $\sigma_m|s = \sigma_v$ for all $m$, i.e. no disclosure by any insider, is an equilibrium if and only if $\sigma_z^2 \geq \sigma_u^2/8$. Other symmetric pure strategy equilibria also exist when $\sigma_z^2 = \sigma_u^2/8$. If $\sigma_z^2 < \sigma_u^2/8$, then there exists an equilibrium where all insiders randomize between $\sigma_v$ and another information advantage in $(0, \sigma_v)$.

(ii) When $M \geq 3$, $\sigma_m|s = \sigma_v$ for all $m$ is the only possible symmetric pure strategy equilibrium and this equilibrium exists only if $\sigma_z^2$ is sufficiently larger than $\sigma_u^2(1/M^2 - 1/M^3)$. When no disclosure is not an equilibrium, there exists an equilibrium where all insiders randomize between $\sigma_v$ and another information advantage in $(0, \sigma_v)$.

Hence, when there is some noise in all assets, equilibrium disclosure policies are such that all insiders retain some information. If, moreover, the volume of noise trading in each asset is sufficiently large, then no disclosure at all is an equilibrium. These results suggest that insider trading and disclosure regulations have more reason to be imposed and enforced in highly liquid markets than in less liquid ones. That is, the competition for discretionary liquidity trades does not suffice to prevent insider trading.

As shown in appendix, the greater the number of assets, the smaller $\sigma_z^2$ has to be for no disclosure to be an equilibrium. That is, given $M$, there exists a $\hat{\alpha}$ such that no disclosure is an equilibrium $\sigma_z^2 \geq \hat{\alpha}$.

\(^1\)Note that there could be asymmetric pure strategy equilibria where some insiders do not disclose any information while others disclose some information.
Insiders determine their disclosure policies ($\sigma^2_{m}$).
Insiders become informed. Public signals $s_1, \ldots, s_M$ are sent.
Traders submit their orders.
Prices are determined by market makers.
Payoffs are realized.

Figure 1: Timing of events.

$\sigma_m \mid s_u m (\sigma_m \mid s, \sigma_v)$ Case I
$\sigma_v \sigma v (\sigma_v)$ Case II

Figure 2: Cases where no disclosure is not an equilibrium.
if and only if $\sigma_z^2 \geq \hat{\alpha}\sigma_u^2$. This $\hat{\alpha}$ does not depend on $\sigma_v$ and decreases as $M$ increases.

**Proposition 3** The greater $M$, the smaller $\sigma_z^2$ has to be for $\sigma_{m|s} = \sigma_v$ for all $m$ to be an equilibrium.

Hence the more diversified the discretionary liquidity trader can be, the more likely insiders are not to disclose any information at all. This results adds to the previous one about noise trading activity since bigger stock exchanges are usually associated with greater noise trading volume. Hence the information provided by firms listed on the New York Stock Exchange (NYSE), for example, would be of lower quality than, say, the information provided by firms listed on the Amsterdam Stock Exchange.

One empirical implication of this result is that the grade portfolio managers and analysts would give to the quality of information provided by publicly traded companies should differ from one country to another depending on the size of their respective stock exchanges. Bhattacharya and Daouk (2002) have gathered data on insider trading regulations from around the world that show a strong positive correlation between the size of the main stock exchange (as measured by either market capitalization or trading volume) of a country and the number of years since insider trading laws have been enacted in that country. Note, moreover, that there is almost no correlation between the number of years since insider trading laws have been voted in a country and the age of the main stock exchange.

Germany, for instance, has had its main stock exchange established in 1585 but its first insider trading law in 1994, whereas the U.S. has had its main stock exchange established in 1792 and its first insider trading law enacted in 1934. When looking at volume and market capitalization of each of these two countries’ main exchange, the U.S. volume is almost three times that of Germany.
and the U.S. market capitalization is nearly eleven times that of Germany. These numbers provide support to my theory since the need for insider trading regulation in the real world seems to arise from the size of the market.

6 Conclusion

A model has been developed where insiders compete to attract the trades of a discretionary liquidity trader by disclosing their private information. In the presence of at least two assets without noise trading (no-noise assets), any equilibrium is such that at least two insiders with no-noise assets fully disclose their private information. Otherwise, the presence of noise trading in at least all assets but one leads to equilibria where all insiders retain some information.

These results have implications regarding the way a public company should be analyzed. As has been shown in this paper, the less frequently traded a company is, the more precise its disclosure statements should be, on average. Hence the manager of a portfolio dealing with small companies with low trading volumes should put more weight on what the management of these companies have to say than managers of portfolios of large companies with high trading volumes.

There has been many remarks in the past regarding the efficacity of the Ontario Securities Commission (OSC) at reducing insider trading on the Toronto Stock Exchange. Market players have often complained that the OSC was not as active as the SEC, in the U.S., in prosecuting traders for insider trading. In defense of the OSC, the results found in this paper suggest that there may be less needs for prosecution in Ontario than in the U.S. since markets in the former are much smaller than the U.S. markets.

The results found here have some empirical implications. It would be interesting to look at
how fund managers analyze their portfolio companies and whether there is a difference between large-cap and small- and micro-cap funds. Also, one could compare the U.S. and Canada in terms of the precision of insiders’ public statements. For example, one could look at the information content of earnings estimate and see if these estimates are more precise for Canadian than American companies.

A Appendix

A.1 Pricing Functions and Insiders’ Expected Profits

Suppose Insider m’s trading strategy is linear in her private signal, i.e.

\[ X_m = \alpha_m + \beta_m V_m, \]

where \( \alpha_m \) and \( \beta_m \) are constant. \( V_m \) and \( S_m \) being multivariate normal, this implies that \( X_m \) given \( s_m \) is also normally distributed with, say, a mean \( \mu_{x|m} \) and a variance \( \sigma_{x|m}^2 \). The aggregate order flow in asset \( m \), \( Y_m = X_m + g_m U + Z_m \), is therefore normally distributed and

\[ E[V_m|s_m, y_m] = E[V_m|s_m] + \lambda_m (y_m - \mu_{x|m}), \]

where

\[ \lambda_m = \frac{\text{cov}(V_m, Y_m|s_m)}{\text{var}(Y_m|s_m)}. \]

Since \( p_m = E[V_m|s_m, y_m], E[U|v_m, s_m] = 0 \) and \( E[Z_m|v_m, s_m] = 0 \), Insider m’s order flow is obtained from

\[ \max_x E \left[ (V_m - (E[V_m|s_m] + \lambda_m (x - \mu_{x|m}))) x|v_m, s_m \right], \]

which yields

\[ x_m = \frac{1}{2\lambda_m} (v_m - E[V_m|s_m] - \lambda_m \mu_{x|m}), \]

where \( E[V_m|s_m] = \frac{\sigma_{x|m}^2}{\sigma_v^2 + \sigma_{x|m}^2}. \)
From last equation, we have
\[ E[X_m|s_m] = \mu_{x|s} = \frac{1}{2}\mu_{x|s}, \]
which holds only if \( \mu_{x|s} = 0 \), and thus
\[ x_m = \frac{1}{2\lambda_m} (v_m - E[V_m|s_m]). \]
Hence \( x_m \) is effectively a linear function of \( v_m \), with \( \beta_m = \frac{1}{2\lambda_m} \) and \( \alpha_m = -\frac{1}{2\lambda_m} E[V_m|s_m] \), and thus our initial conjecture was correct.

It is now possible to find the distribution of \( X_m \) given \( s_m \), which is
\[ X_m|s_m \sim N\left(0, \frac{\sigma^2_{m|s}}{4\lambda^2_m}\right), \]
where \( \sigma^2_{m|s} = \text{var}(V_m|s_m) = \frac{\sigma^2_v \sigma^2_m}{\sigma^2_v + \sigma^2_m} \). Then
\[ \lambda_m = \frac{\text{cov}(V_m, X_m|s_m)}{\text{var}(Y_m|s_m)} = \frac{\text{cov}(V_m, X_m|s_m)}{\text{var}(Y_m|s_m)} = \frac{1}{2\lambda_m} \frac{\sigma^2_{m|s}}{\sigma^2_v + \sigma^2_u + \sigma^2_m}, \]
which gives us
\[ \lambda_m = \frac{\sigma_{m|s}}{2} \frac{2\sigma_u^2}{\sigma_u^2 + \sigma_m^2}. \]
Insider \( m \)'s expected payoff before knowing \( v_m \) is then
\[ \frac{1}{2} \sigma_{m|s} \sqrt{g_m^2 \sigma_u^2 + \sigma_m^2}. \]

A.2 Proof of Lemma 1

1. If \( \sigma_{m|s} = 0 \) for some \( m \), then trading in these assets is costless for the discretionary liquidity trader.

2. Suppose \( \sigma_{m|s} > \sigma_{m'|s} \). If \( g_m > 0 \), then \( \lambda_k g_k = \frac{\sigma_{m|s}}{2\sigma_u^2} \) for all \( k \) with \( g_k > 0 \), which implies that \( g_{m'} = 0 \).

Note that \( \frac{\sigma_{m|s}}{2\sigma_u} \) is then the liquidity trader’s expected loss. If a market maker were to post a pricing function \( p_{m'} = E[V_{m'}|S_{m'}] + \frac{\sigma_{m'}^2}{2\sigma_u^2} y_{m'} \), where \( \sigma_s \in (\sigma_{m'|s}, \sigma_{m|s}) \), he could correctly anticipate \( g_{m'} = 1 \) since the liquidity trader’s expected loss would then be \( \frac{\sigma_{m|s}}{2\sigma_u^2} \), which is smaller than \( \frac{\sigma_{m|s}}{2\sigma_u^2} \). This market
From Lemma 1, we know that $\sigma_1$ always be willing to choose $\hat{\sigma}$, thus $\hat{\sigma}_u$ and thus $\Sigma_u$ such that $\sum_m \frac{\sigma_m^2}{\sigma_m^2 + \gamma^2} = 1$.

A.3 Proof of Proposition 1

1. Consider first pure strategy equilibria. Let $\sigma_m|s = \underline{\sigma}_s > 0$ for all $m \in M$. From Lemma 1, we know that $g_m > 0$ only if $m \in M$. Hence a vector $\Sigma_s$ such that $\sigma_m'|s > \underline{\sigma}_s$ for some $m'$ cannot be an equilibrium since then $g_m' = 0$, and thus $u_m' = 0$, whereas any $\sigma_m'|s \in (0, \underline{\sigma}_s)$ would yield $g_m' = 1$ and thus $u_m' > 0$.

A vector $\Sigma_s$ such that $\sigma_m|s = \underline{\sigma}_s > 0$ for all $m$ cannot be an equilibrium either. In this case, there exists an asset, $m'$, say, such that $g_m' < 1$. With an information advantage of $\sigma_s - \varepsilon$ instead of $\underline{\sigma}_s$, where $\varepsilon \in (0, \underline{\sigma}_s)$, Insider $m'$'s expected profit would be $\sigma_s^2 (\underline{\sigma}_s - \varepsilon)$ since then asset $m'$ would have all the discretionary liquidity trades. With $\varepsilon$ sufficiently small, Insider $m$ is better off choosing $\sigma_s - \varepsilon$ instead of $\underline{\sigma}_s$ and thus $\Sigma_s$ cannot be an equilibrium. Hence a pure strategy equilibrium where $\sigma_m|s > 0$ for all $m$ does not exist.

Consider now a mixed strategy profile $\theta$ such that no insider plays $\sigma_m|s = 0$ with certainty. Take two assets, $m'$ and $m''$, and let $\theta_m'$ and $\theta_m''$ denote the strategy of insider $m'$ and $m''$, respectively. Let also $\hat{\sigma}_i|s$ be the smallest $\sigma_i|s \in A_i$ such that $\int_0^{\hat{\sigma}_i|s} d\theta_i = 1$, $i = m', m''$. That is, $\hat{\sigma}_i|s$ denotes the upper bound to all $\sigma_i|s$'s that can arise under $\theta_i$. Using an argument similar to the above, Insider $m'$ will always be willing to choose $\hat{\sigma}_m'|s < \hat{\sigma}_m''|s$, and vice versa, until $\hat{\sigma}_m'|s = \hat{\sigma}_m''|s = 0$. Hence there are no mixed strategy equilibria where all insiders retain some information either.
2. Let \( \hat{\sigma}_s = \min \{ \sigma_{m|s} : \sigma_{z,m} > 0 \} \) and suppose that asset \( m' \) is such that \( \sigma_{z,m'} = 0 \). If \( \sigma_{m'|s} \geq \hat{\sigma}_s \), then \( g_{m'} = 0 \) and \( u_{m'} = 0 \) while it is possible for Insider \( m' \) to choose \( \sigma_{m'|s} > 0 \) sufficiently small to have \( g_{m'} > 0 \) and thus \( u_{m'} > 0 \). Therefore, in equilibrium Insider \( m' \) will disclose more information than all insiders whose assets are traded by noise traders.

### A.4 Proof of Proposition 2

(i) Given a vector a information asymmetry \( \Sigma_s \), we have

\[
\frac{du_m(\Sigma_s)}{d\sigma_{m|s}} = \frac{\sigma_s \sigma_{m|s}^3}{2 \left( \sigma_{m|s}^2 - \hat{\sigma}_s^2 \right)^{3/2}} \left( 1 + \frac{\sigma_u^3 g_m^3}{\sigma_u^2 + \sigma_u^2 \bar{\Sigma}_{k=1}^M g_k^3 - \sigma_u^2 g_m^3 + \sigma_u^2} \right).
\]

Let \( (\sigma_{m|s}, \bar{\Sigma}_{s,-m}) \) be the vector \( \Sigma_s \) where \( \sigma_{k|s} = \hat{\sigma}_s > 0 \) for all \( k \neq m \). When \( \Sigma_s = (\sigma_{m|s}, \bar{\Sigma}_{s,-m}) \),

\[
g_k = (1 - g_m)/(M - 1) \text{ for all } k \neq m \text{ and thus}
\]

\[
\frac{du_m(\sigma_{m|s}, \bar{\Sigma}_{s,-m})}{d\sigma_{m|s}} = \frac{\sigma_s \sigma_{m|s}^3}{2 \left( \sigma_{m|s}^2 - \hat{\sigma}_s^2 \right)^{3/2}} \left( 1 + \frac{\sigma_u^3 g_m^3}{\sigma_u^2 + \sigma_u^2 \bar{\Sigma}_{m|s}^3 + \sigma_u^2 (1 - g_m)^3 - \sigma_u^2 g_m^3 + \sigma_u^2} \right).
\]

Replacing \( \hat{\gamma} \) by \( \frac{\sigma_{m|s}^3}{\sigma_u^2 + \sigma_u^2 \bar{\Sigma}_{m|s}^3 + \sigma_u^2} \), the last equation becomes

\[
\frac{du_m(\sigma_{m|s}, \bar{\Sigma}_{s,-m})}{d\sigma_{m|s}} = \frac{(\sigma_u^2 g_m^3 + \sigma_u^3)^{3/2}}{2\sigma_u^2 g_m^3} \left( 1 + \frac{\sigma_u^3 g_m^3}{\sigma_u^2 + \sigma_u^2 \bar{\Sigma}_{m|s}^3 + \sigma_u^2 (1 - g_m)^3 - \sigma_u^2 g_m^3 + \sigma_u^2} \right).
\]

When \( \sigma_{m|s} = \hat{\sigma}_s \), \( g_m = \frac{1}{M} \) and then

\[
\frac{du_m(\hat{\sigma}_s, \bar{\Sigma}_{s,-m})}{d\sigma_{m|s}} = \frac{(\sigma_u^2 g_m^3 + \sigma_u^3)^{1/2}}{2\sigma_u^2 g_m^3} \left( \sigma_u^2 + \sigma_u^2 \left( \frac{1}{M^3} - \frac{1}{M^2} \right) \right),
\]

so we need \( \sigma_u^2 \geq \sigma_u^2 (1/M^2 - 1/M^3) \) to have \( \frac{du_m(\hat{\sigma}_s, \bar{\Sigma}_{s,-m})}{d\sigma_{m|s}} \geq 0 \).

To have a maximum when \( du_m/d\sigma_{m|s} = 0 \), we need the second-order condition

\[
d^2 u_m / (d\sigma_{m|s})^2 = \frac{d}{dg_m} \frac{du_m}{d\sigma_{m|s}} \times \frac{dg_m}{d\sigma_{m|s}} \leq 0.
\]

Since \( \frac{dg_m}{d\sigma_{m|s}} \leq 0 \), this condition translates into

\[
\frac{d}{dg_m} \frac{du_m}{d\sigma_{m|s}} \geq 0.
\]
In the present case, we have

\[
\frac{d}{dg_m} \frac{du_m(\sigma_{m|s}, \Sigma_{s,-m})}{d\sigma_m|s} = \frac{3\sigma^2 m (\sigma^2 m + \sigma^2 s)^{3/2}}{2\sigma^2 s} \left( 1 + \frac{\sigma^2 m}{\sigma^2 s + \sigma^2 m + \sigma^2 (1 - \sigma_m^2)} - \frac{2\sigma^2 m}{\sigma^2 s + \sigma^2 m} \right) + \frac{(\sigma^2 m \sigma^2 s + \sigma^2 m)^{3/2}}{2\sigma^2 s} \left( 3\sigma^2 m \sigma^2 m + \sigma^2 m^3 + \sigma^2 (1 - \sigma_m^2) \right) - \frac{4\sigma^2 m (\sigma^2 m + \sigma^2 s)^2 - 4\sigma^2 m^3}{(\sigma^2 s + \sigma^2 m)^2}.
\]

If \( \sigma_{m|s} = \bar{s} \) and \( \frac{du_m(\sigma_{s}, \Sigma_{s,-m})}{d\sigma_m|s} = 0 \), then

\[
\frac{d}{dg_m} \frac{du_m(\bar{s}, \Sigma_{s,-m})}{d\sigma_m|s} = \frac{\sigma^2 u \left( \frac{3\sigma^2 u}{M} - \frac{3\sigma^2 u}{M^2} - 4\sigma^2 z \right)}{M(\sigma^2 s + \sigma^2 m/M^2)^2}.
\]

To have \( d^2 u_m(\sigma_s, \Sigma_{s,-m})/(d\sigma_{m|s})^2 \leq 0 \), we need \( \frac{3\sigma^2 u}{M} + \frac{3\sigma^2 u}{M^2} - 4\sigma^2 z \geq 0 \). Since \( du_m(\sigma_{s}, \Sigma_{s,-m})/d\sigma_{m|s} = 0 \) implies that \( \sigma^2 z = \sigma^2 u (1/M^2 - 1/M^3) \), a strategy profile \((\bar{s}, \Sigma_{s,-m})\) can be an equilibrium in this case only if

\[
3\sigma^2 u (1/M^2 - 1/M^4) + 3\sigma^2 u/M^3 - 4\sigma^2 u (1/M^2 - 1/M^3) = \frac{\sigma^2 u}{M^4} (10M - 3 - 4M^2) \geq 0,
\]

which is not possible when \( M > 2 \). Hence a symmetric pure strategy equilibrium does not exist when \( M \geq 3 \) and \( \sigma^2 z = \sigma^2 u (1/M^2 - 1/M^3) \).

Let \( \sigma^2 z = \alpha \sigma^2 u \). Then \( du_m(\sigma_{m|s}, \Sigma_{s,-m})/d\sigma_{m|s} \) can be rewritten as

\[
\frac{du_m(\sigma_{m|s}, \Sigma_{s,-m})}{d\sigma_{m|s}} = \frac{\sigma^2 u g^2 m + \alpha^3/2}{2\alpha} \left( \alpha - \frac{g^2 m}{\sigma^2 m + \alpha} + \frac{g^3 m}{\alpha + \sigma^3 m + (1 - \sigma_m)^2} \right).
\]

If \( \bar{s} = \sigma_v \) and Insider \( m \) chooses \( \sigma_{m|s} \neq \sigma_v \), then \( g_m \) is greater than \( \frac{1}{M} \) since then Insider \( m \) discloses more information than all the other insiders. Hence an insider has no incentive to deviate from the no-disclosure outcome when \( \frac{du_m(\sigma_{m|s}, \Sigma_{s,-m})}{d\sigma_{m|s}} > 0 \) for all \( g_m \in (\frac{1}{M}, 1) \).

When \( M = 2 \), \( A_m > 0 \) for all \( \alpha \geq 1/M^2 - 1/M^3 = 1/8 \) and all \( g_m \in (\frac{1}{2}, 1] \) and thus Insider \( m \)'s best reply to \( \sigma_{k|s} = \sigma_v, k \neq m \), is always \( \sigma_{m|s} = \sigma_v \) when \( \sigma^2 z \geq \sigma^2 u/8 \). When \( \sigma^2 z = \sigma^2 u/8 \), there are other symmetric pure strategy equilibria, as long as the information asymmetry chosen by each insider is not too small.
(ii) When $M \geq 3$, $\sigma_z^2$ has to be sufficiently larger than $\sigma_z^2(1/M^2 - 1/M^3)$ to ensure that $\sigma_{m|s} = \sigma_v$ for all $m$ be an equilibrium. Note, however, that $\frac{\alpha-\sigma_{m|s}^2}{g_m^\alpha} + \frac{\sigma_{m|s}^2}{\alpha+g_m^\alpha + \frac{(1-g_m)^3}{(M-1)^2}} > 0$ for all $g_m \in [\frac{1}{M}, 1]$, all $\alpha \geq 1/8$ and all $M \geq 3$. That is, the no-disclosure outcome $\sigma_{m|s} = \sigma_v$ for all $m$ is an equilibrium whenever $\sigma_z^2 \geq \sigma_v^2/8$ for all $M \geq 3$ also. Moreover, since $\frac{1/8-g_m^2}{g_m+1/8} + \frac{g_m^2}{1/8+g_m^2 + \frac{(1-g_m)^3}{(M-1)^2}} > 0$ for all $M \geq 3$ when $g_m = \frac{1}{M}$, there exists a $\sigma_z^2 < \sigma_v^2/8$ such that the no-disclosure equilibrium exists whenever $M \geq 3$.

That is, with three or more assets, the no-disclosure equilibrium exists for smaller values of $\sigma_z^2$ than with two assets.

Suppose now that a symmetric pure strategy equilibrium does not exist. Let then $\hat{\theta}^p = (\hat{\theta}^p_m, \hat{\theta}^-m)$ denote the strategy profile where all insiders choose $\sigma_v$ with probability $p$ and some $\hat{\sigma}_s \in (0, \sigma_v)$ with probability $1-p$. For $\hat{\theta}^p$ to be an equilibrium, we need

$$\frac{du_m(\hat{\sigma}_s, \hat{\theta}^-m)}{d\sigma_{m|s}} = 0 \quad \text{and} \quad u_m(\hat{\sigma}_s, \hat{\theta}^-m) = u_m(\sigma_v, \hat{\theta}^-m).$$

(6)

To construct such an equilibrium, take $\hat{\sigma}_s$ such that $u_m(\sigma_s(\hat{\Sigma}_{s,-m}), \hat{\Sigma}_{s,-m}) < u_m(\sigma_v, \hat{\Sigma}_{s,-m})$, where $\hat{\Sigma}_{s,-m}$ is an $(M-1) \times 1$ vector of $\hat{\sigma}_s$’s and $\sigma(\hat{\Sigma}_{s,-m})$ is such that $\frac{du_m(\sigma(\hat{\Sigma}_{s,-m}), \hat{\Sigma}_{s,-m})}{d\sigma_{m|s}} = 0$ (note that $\sigma(\hat{\Sigma}_{s,-m}) < \hat{\sigma}_s$). Such a $\hat{\sigma}_s$ exists since

$$u_m(\sigma(\hat{\Sigma}_{s,-m}), \hat{\Sigma}_{s,-m}) < \frac{\hat{\sigma}_s}{2} \sqrt{\sigma_a^2 + \sigma_z^2} \leq \frac{\sigma_v \sigma_z}{2} \leq u_m(\sigma_v, \hat{\Sigma}_{s,-m})$$

whenever $\hat{\sigma}_s \leq \frac{\sigma_v \sigma_z}{\sqrt{\sigma_a^2 + \sigma_z^2}}$. On the other hand, the non-existence of a non-disclosure equilibrium implies that

$$u_m(\sigma(\sigma_v), \sigma_v) > u_m(\sigma_v, \sigma_v).$$

Let $\check{\sigma}_s$ be such that $u_m(\sigma(\hat{\Sigma}_{s,-m}, \hat{\Sigma}_{s,-m}) = u_m(\sigma_v, \hat{\Sigma}_{s,-m})$. Then, for any $\hat{\sigma}_s \in (0, \check{\sigma}_s)$, we have that

$$u_m(\sigma(\hat{\theta}^0_m), \hat{\theta}^0_m) = u_m(\sigma(\hat{\Sigma}_{s,-m}), \hat{\Sigma}_{s,-m}) < u_m(\sigma_v, \check{\Sigma}_{s,-m}),$$

and

$$u_m(\sigma(\hat{\theta}^1_m), \hat{\theta}^1_m) = u_1(\sigma(\sigma_v), \sigma_v) > u_m(\sigma_v, \sigma_v).$$

Therefore, since $u_m(\sigma(\hat{\theta}^p_m), \hat{\theta}^p_m)$ is continuous in $p$, there exists a $p, \hat{p}$, say, such that $u_m(\sigma(\hat{\theta}^-m), \hat{\theta}^-m) = u_m(\sigma_v, \hat{\theta}^-m)$. If $\sigma(\hat{\theta}^-m) = \hat{\sigma}_s$, then $\hat{\theta}^p$ is an equilibrium.
A value $\hat{\sigma}_s \in (0, \hat{\sigma}_s)$ such that $\sigma(\hat{\phi}_m) = \hat{\sigma}_s$ always exists: If, for instance, $\hat{\sigma}_s$ is sufficiently close to zero, then $\sigma(\hat{\phi}_2)$ is close to $\sigma_1(\sigma_v)$ and thus is greater than $\hat{\sigma}_s$; if, on the other hand, $\hat{\sigma}_s$ is sufficiently close $\hat{\sigma}_s$, then $u_m(\sigma(\hat{\sigma}_s), \hat{\Sigma}_{s,-m}) \approx u_m(\sigma_v, \hat{\Sigma}_{s,-m})$, in which case $\hat{p}$ is close to zero and thus $\sigma(\hat{\phi}_2) < \hat{\sigma}_s$. The derivative $du_m(\sigma, \hat{\Sigma}_{s,-m})/d\sigma$ being continuous in $\sigma$, the function $\sigma(\hat{\phi}_2)$ is continuous in both $p$ and $\hat{\sigma}_s$ and thus there exists a $\hat{\sigma}_s \in (0, \hat{\sigma}_s)$ such that $\sigma(\hat{\phi}_2) = \hat{\sigma}_s$.

A.5 Proof of Proposition 3

It can be shown numerically that the greater $M$, the smaller $\sigma^2_z$ has to be in order for the no-disclosure outcome to be an equilibrium. To explain how this can be shown, first note that, using (4), we can write

$$\sigma_m(\sigma_v) = \sqrt{\frac{\sigma^2_v (\sigma^2_u + \sigma^2_v/g^2_m)}{\sigma^2_u + \sigma^2_v (1-g_m/M-1)^{-2}}},$$

and thus

$$u_m(\sigma_m(\sigma_v), \sigma_v) = \frac{1}{2} \sqrt{\frac{\sigma^2_v (\sigma^2_u + \sigma^2_v/g^2_m)}{\sigma^2_u + \sigma^2_v (1-g_m/M-1)^{-2}}} \times \sqrt{\sigma^2_u g^2_m + \sigma^2_z}.$$

For any admissible $\alpha$, let $g_m(\alpha)$ be such that $\frac{\alpha-g_m(\alpha)^2}{\alpha+g_m(\alpha)^2} + \frac{g_m(\alpha)^2 (1-g_m(\alpha)^2)}{(1-M)^2} = 0$. This gives us

$$u_m(\sigma_m(\sigma_v), \sigma_v) = \frac{\sigma_v}{2} \sqrt{\frac{1+\alpha/g_m(\alpha)^2}{1+\alpha (1-g_m(\alpha)/M-1)^{-2}}} \times \sigma_u \sqrt{g_m(\alpha)^2 + \alpha}.$$

Given $\sigma^2_z$ and $M$, let then $\hat{\sigma}$ be such that $u_m(\sigma_m(\sigma_v), \sigma_v) = u_m(\sigma_v, \sigma_v)$, i.e.

$$\frac{\sigma_v}{2} \sqrt{\frac{1+\alpha/g_m(\alpha)^2}{1+\alpha (1-g_m(\alpha)/M-1)^{-2}}} \times \sqrt{g_m(\alpha)^2 + \alpha} = \frac{\sigma_v}{2} \sqrt{1/M^2 + \hat{\sigma}}.$$

Note that $\hat{\sigma}$ is unique and does not depend on $\sigma_v$. In this case, we have that no disclosure is an equilibrium whenever $\sigma^2_z \geq \hat{\sigma} \sigma^2_u$ regardless of $\sigma_v$. If $\hat{\sigma}$ is associated with $M = N$, say, it can then be verified that

$$\frac{\alpha-g_m(\alpha)^2}{\alpha+g_m(\alpha)^2} + \frac{g_m(\alpha)^2 (1-g_m(\alpha)^2)}{N^2} > 0$$

for all $g_m \in \left[\frac{1}{N+1}, 1\right]$, and this for all $N \geq 3$. It therefore exists a $\sigma^2_z < \hat{\sigma} \sigma^2_u$ such that no disclosure is an equilibrium when $M = N + 1$. 
References


