International Parity Conditions

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Outline of the Chapter

How are exchange rates determined? Can we predict them?

• Prices and Exchange Rates
  – Prices Indices
  – Inflation Rates

• Interest Rates and Exchange Rates
  – Absence of Arbitrage Opportunities
Prices and Exchange Rates

An exchange rate is the price of a currency in terms of another currency.

What do we usually do with our currencies? Buy goods and services.

Prices of G&S derived from supply and demand are equilibrium prices.

Prices and Exchange Rates

If G&S can freely be exchanged between countries, then identical goods should have the same price in every country.

G&S that sell for different prices in two different countries should then create some pressure on the exchange rate of these countries’ currencies.

Equilibrium Prices  ⇒  Equilibrium Exchange Rates?
In frictionless markets,

\[ p^\$ \times s = p^¥, \]

where \( s \) is the exchange in \( ¥/\$ \), \( p^\$ \) and \( p^¥ \) are prices in the U.S. and Japan, respectively.

**Purchasing power parity (PPP) exchange rate:**

\[ s = \frac{p^¥}{p^\$} \]

In theory, the law of one price should hold for *all* goods and services.

Comparing identical goods in different markets should give us equilibrium exchange rates.

**Absolute version of the PPP theory:** Spot exchange rates are determined by the relative prices of similar baskets of goods.
The Hamburger Standard

Let’s apply the PPP theory to Big Mac prices.

Price of a Big Mac: C$ 3.33 and US$2.49 (April 25, 2002).

**PPP exchange rate:** C$3.33/US$2.49 = C$1.34/US$.

**Actual exchange rate:** C$1.57/US$.

Undervaluation of the C$ (on the spot market):

\[
\frac{\text{PPP Exchange Rate} - \text{Spot}}{\text{Spot}} = \frac{1.34}{1.57} - 1 = -15\%.
\]

In a less extreme form, the PPP theory should hold for baskets of goods.

\[ PI^¥ \equiv \text{price of a basket of goods in yen} \]
\[ PI^$ \equiv \text{price in US$ of the same basket} \]

PPP exchange rate between Japan and US:

\[ S = \frac{PI^¥}{PI^$} \]
Relative Purchasing Power Parity

Relative changes in prices between two countries over a period of time is what determines the changes in exchange rates over the same period.

If the spot exchange rate between two countries starts in equilibrium, any change in the differential rate of inflation between them tends to be offset over the long run by an equal but opposite change in the spot exchange rate.

$PI_t^i \equiv \text{Price index in Country } i \text{ at time } t, \ i = \¥, \$$

$\pi^i \equiv \text{inflation in Country } i \text{ from time } t \text{ to } t + 1$

$S_t \equiv \text{exchange rate at time } t \text{ in } \¥/\$

Assuming PPP theory holds at time $t$,

$$S_t = \frac{PI_t^¥}{PI_t^\$} \quad \text{and} \quad S_{t+1} = \frac{PI_t^¥(1 + \pi^¥)}{PI_t^\$(1 + \pi^\$)} = S_t \times \frac{1 + \pi^¥}{1 + \pi^\$}$$
Relative Purchasing Power Parity

Percent change in the value of the yen from $t$ to $t + 1$:

\[
\frac{1}{S_{t+1}} - \frac{1}{S_t} = \frac{S_t}{S_{t+1}} - 1
\]

\[
= \frac{S_t}{S_t \times \frac{1 + \pi^{¥}}{1 + \pi^{¥}} - 1}
\]

\[
= \frac{1 + \pi^{¥}}{1 + \pi^{¥}} - 1
\]

\[
= \frac{\pi^{¥} - \pi^{¥}}{1 + \pi^{¥}} \approx \pi^{¥} - \pi^{¥}
\]

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Relative Purchasing Power Parity: An Example

Honda’s price per vehicle: ¥4,000,000

\[S = ¥125/¥\]

Expected inflation in Japan: 1%

Expected inflation in U.S.: 3%

Assuming PPP, what is the expected spot exchange rate at the end of the year?
Relative Purchasing Power Parity: An Example

\[ S_{\text{end}} = S \times \frac{1 + \pi^¥}{1 + \pi^\$} \]

\[ = ¥125/$ \times \frac{1.01}{1.03} \]

\[ = ¥122.57/$ \]

PPP and Transaction Costs

Helliwell (1998) Canadian provinces are 12 times more likely to trade merchandises and 40 times more likely to trade services among themselves then with closer American states

The border matters
Tests of Purchasing Power Parity

PPP holds well over the very long run but poorly for shorter time periods.

The theory holds better for countries with relatively high rates of inflation and underdeveloped capital markets.

Example

Businesses and consumers tend to abandon their currency in a hyperinflationary economy.

Suppose a TV set is priced 1,200 pesos in Mexico, reflecting a $400 TV price and 3 peso/$. If the peso devalues to 6 peso/$, the price of a TV set becomes 2,400 pesos.

PPP holds immediately since merchants use PPP to calculate their prices.
Real and Nominal Exchange Rates

Suppose PPP holds:

\[ \frac{\$1.50/\£}{\£1,500/\text{U.S. good}} = \frac{\£1,000/\text{British good}}{\frac{\$1,500}{\£1,000}} \]

and thus

\[ \frac{\$1.50/\£}{\£1,500/\£1,000} = 1 \text{ British good per U.S. good,} \]

i.e. identical goods trade on a one-for-one basis.
Prices and Exchange Rates

Real and Nominal Exchange Rates

Real Exchange Rate = \frac{\text{Nominal Exchange Rate}}{\text{PPP Exchange Rate}}

\[ S_r(\$/\£) = \frac{S_n(\$/\£)}{PI^$/PI^\£} \]

If \( S_r(\$/\£) > 1 \), US$ is undervalued vis-à-vis the £.

Prices and Exchange Rates

Real and Nominal Exchange Rates

Real Exchange Rate Index = \frac{\text{Nominal Exchange Rate Index}}{\text{PPP Exchange Rate Index}}

\[ E_r(FC/$) = \frac{E_n(FC/$)}{C^{FC}/C^$} \]

If changes in exchange rates offset differential inflation, \( E_r = 100 \).
Exchange Rate Pass-Through

Incomplete exchange rate pass-through could explain why an exchange rate can deviate from its PPP-equilibrium level. This may depend on demand elasticities:

\[ \varepsilon_p = \frac{\Delta Q_p}{\Delta p} \]

Goods with inelastic demand, i.e. \(|\varepsilon_p| < 1\), should be associated with a higher degree of pass-through.

Exchange Rate Pass-Through: An Example

BMW produces automobiles in Germany and pays all production expenses in euros. When exporting to the US, the price should be

\[ P_{\text{BMW}}^\$ = P_{\text{BMW}}^\text{€} \times S \]

If the euro appreciates by 10% over the US$, then \( P_{\text{BMW}}^\$ \) should also increase by 10%.

If, instead, \( P_{\text{BMW}}^\$ \) increases by less than 10%, the pass-through is partial. BMW may want to limit the pass-through.
Prices and Exchange Rates

Exchange Rate Pass-Through: Example Continued

Suppose exchange rate is $1/€.

\[ P_{\text{BMW}}^\$ = P_{\text{BMW}}^€ \]

If the euro appreciates 20% versus the dollar and \( P_{\text{BMW}}^€ = €35,000 \),

\[ P_{\text{BMW}}^\$ = 1.2 \times 35,000 = $42,000. \]

Prices and Exchange Rates

Exchange Rate Pass-Through: Example Continued

If, instead, \( P_{\text{BMW}}^\$ = $40,000 \), the pass-through is

\[
\frac{(P_{\text{BMW}}^\$,2 - P_{\text{BMW}}^\$,1)/P_{\text{BMW}}^€_{\text{BMW},1}}{(S_2 - S_1)/S_1} = \frac{(40 - 35)/35}{.20} = \frac{.14}{.20} = 70\%
\]
Prices and Exchange Rates

Exchange Rate Pass-Through: Another Example

Honda’s price per vehicle is ¥4,000,000, $S_b = ¥125/\$

Expected inflation in Japan: 1%, Expected inflation in U.S.: 3%

Prices and Exchange Rates

Exchange Rate Pass-Through: Example Continued

(a) Assuming PPP and 100% pass through, what is the expected price of a Honda in US$ at the end of the year ($e \equiv \text{end}, b \equiv \text{beginning}$)?

Let $\gamma^¥$ denote the percentage change in the value of the yen, i.e.

$$\gamma^¥ = \frac{S_b - S_e}{S_e} \implies S_e = \frac{S_b}{1 + \gamma^¥}$$

(note that we calculate the percentage change this way because the exchange rate is quoted in ¥/$).
Exchange Rate Pass-Through: Example Continued

(a) Assuming PPP, we have

\[ P_b^S \times S_b = P_b^¥ = P_e \times S_e = P_e^¥ \]

\[ P_e^S \times \frac{S_b}{1 + \gamma^¥} = (1 + \pi^¥) P_b^¥ \]

\[ P_e^S = (1 + \gamma^¥)(1 + \pi^¥) \frac{P_b^¥}{S_b} \]

\[ P_e^S = (1 + \gamma^¥)(1 + \pi^¥) P_b^S \]

That is, the increase in the US price is the compounding effect of inflation in Japan and the percentage appreciation of the yen.

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In the present case, we have

\[ S_b = ¥125/$, \quad S_e = \frac{1.01 \times 125}{1.03} = ¥122.57/$ \]

and thus the Japanese yen appreciates by

\[ \frac{1}{1/S_e} - \frac{1}{S_b} = \frac{S_b - S_e}{1/S_b} = \frac{125 - 122.57}{122.57} = 1.98\% \]

versus the dollar.
Prices and Exchange Rates

Exchange Rate Pass-Through: Example Continued

(a) If the exchange rate pass-through is 100%, then the new price of a Honda in the US is (the beginning price is 4,000,000/125 = $32,000)

\[
P_e^S = (1 + 100\% \times \gamma^¥)(1 + \pi^¥)P_b^S = 1.0198 \times 1.01 \times 32,000 = 32,960.
\]

(b) Assuming PPP and 60% pass through, the new price of a Honda in US$ at the end of the year is

\[
P_e^S = (1 + 60\% \times \gamma^¥)(1 + \pi^¥)P_b^S = 1.0119 \times 1.01 \times 32,000 = 32,704.
\]
Interest Rates and Exchange Rates

The Fisher Effect

\[ 1 + r = \frac{1+i}{1+\pi} \Rightarrow i = r + \pi + r\pi \]

- \( r \equiv \) real exchange rate
- \( i \equiv \) nominal exchange rate
- \( \pi \equiv \) inflation rate

The International Fisher Effect

- \( i^S \equiv \) nominal interest in the US
- \( i^¥ \equiv \) nominal interest rate in Japan
- \( S_1 \equiv \) exchange rate (in ¥/$) at the beginning of the period
- \( S_2 \equiv \) exchange rate at the end of the period

Return should be the same in both markets

\[ \frac{S_1}{S_2} (1 + i^¥) = 1 + i^S \Rightarrow \frac{S_1}{S_2} = \frac{1 + i^S}{1 + i^¥} \]

Subtracting one on both sides:

\[ \frac{S_1 - S_2}{S_2} = \frac{i^S - i^¥}{1 + i^¥} \]
The International Fisher Effect

Empirical tests provide some support to this thesis. This model does not take into account foreign exchange risk. Speculation may also create distortions in currency markets.

The Forward Rate

The forward rate is an exchange rate quoted for settlement at some future date: 30, 60, 90, 180, 270, 360 days.

Invest $1 in the U.S. at the rate $i$ and enter a forward contract that exchanges US$ for Swiss Francs (SF) at the rate $F_{SF/\$}$. Exchange US$ for SF today at the spot rate $S_{SF/\$}$ and invest it in Switzerland at the rate $i_{SF}$. 
The Forward Rate

Should end up with the same number of SF:

\[
\left(1 + i^\text{SF/\$}\right) F^{\text{SF/\$}} = \left(1 + i^{\text{SF/\$}}\right) S^{\text{SF/\$}}
\]

\[F_{90}^{\text{SF/\$}} \equiv 90\text{-day forward rate for the SF/\$ exchange rate}\]

\[i^c \equiv \text{annual interest rate in (currency } c\text{)-denominated deposits}\]

\[
F_{90}^{\text{SF/\$}} = S^{\text{SF/\$}} \times \frac{1 + i^{\text{SF/\$}} \times \frac{90}{360}}{1 + i^{\text{\$/\$}} \times \frac{90}{360}}
\]
Interest rate parity

Spot and forward rates are considered to be at interest rate parity if

\[
\frac{F_n^{SF/\$}}{S^{SF/\$}} = \frac{1 + i^{SF} \times \frac{n}{360}}{1 + i^{\$} \times \frac{n}{360}}
\]

Forward premium:

\[
f_n^{SF/\$} = \frac{S^{SF/\$} - F_n^{SF/\$}}{F_n^{SF/\$}} = \frac{(i^{\$} - i^{SF}) \times \frac{n}{360}}{1 + i^{\$} \times \frac{n}{360}}
\]

Currency with a lower interest rate sells forward at a premium
Covered Interest Arbitrage (CIA)

If spot and forward rates are not at interest rate parity, this creates *covered interest arbitrage* opportunities.

These opportunities will continue until spot and forward rates reach interest rate parity.

Political risk: Application of capital controls

Uncovered Interest Arbitrage (UIA)

Money is borrowed from countries with low interest rates and invested in countries with high interest rates, without covering with a forward contract.

Expected profits calculated using expected spot rates: foreign exchange risk.
Uncovered Interest Arbitrage (UIA)

**Example:** The “yen carry trade”

Borrow yen at extremely low interest rate (0.4%)

Change for dollars, invest the dollars at 5%

Change dollars for yen to repay loan.

If the Yen strengthens by more than the interest rate differential, investors lose money

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Forward Rate as a Predictor of Future Spot Rate

Hypothesis:

\[ E_t[S_{t+k}] = F_{t,t+k} \]

Future spot rates should be distributed around the forward rate if hypothesis is true.

If markets are not efficient, then the forward rate is not an unbiased predictor of the future spot rate.