Tariff under a Nonlinear Tax System in a Small Open Economy

Kam Yu*
Department of Economics
Lakehead University

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Abstract

This paper considers the trade policy of a small open economy under a nonlinear tax system. The government uses progressive taxes to redistribute income from the skilled workers to the unskilled workers. It, however, only observe the workers' income but not their types. Under free trade, the economy imports unskilled labour-intensive goods and export skilled labour intensive goods. We show that when the incentive compatibility constraint for the skilled workers is binding, it is Pareto improving to impose a tariff. This result conforms with the observed trade policies in agricultural goods in many countries.

Keywords: International trade, tariffs, nonlinear tax, income redistribution, asymmetric information.

1 Introduction

Classical trade theory asserts that for a small open economy, the optimal tariff for any imported commodity is zero. Therefore governments should in theory implement a free trade policy. In practice, however, most governments still maintain tariffs on selected goods. Recently there has been a vast literature on strategic trade theory that explains protective trade policies resulting from imperfect competition. For example, Krugman (1984) suggests that some markets are both oligopolistic and

*Kam Yu, Department of Economics, Lakehead University, 955 Oliver Road, Thunder Bay, Ontario, Canada P7B 5E1. Phone: 807-343-8229, Fax: 807-346-7936, e-mail: Kam.Yu@Lakeheadu.ca

1See, for example, Dixit and Norman (1980), p. 159.
segmented. Firms are aware that their actions affect prices and they can charge different prices in different countries. The technology of the industry in question exhibits economy of scale, which can be in forms of static economy of scale, learning-by-doing, or competition in R&D. In each case the government has the incentive to impose a tariff on the foreign firm so that the domestic firm can exploit the scale economy. It charges high price in the domestic market and use the pure profit to cover fixed costs, to move down the learning curve, or to engage in R&D. This makes the domestic firm achieve a lower marginal cost and become more competitive in the international market. Even if a domestic firm does not exist, the government can impose a tariff on the import sold by oligopolistic foreign firms, which can be viewed as a form of rent seeking.\textsuperscript{2}

The above arguments, however, are more appropriate for the high-tech industry. They do not apply to unskilled-labour intensive sectors such as agriculture, where economy of scale and oligopolistic behaviour are not observed. For example, Japan imposes high tariff rates or uses health regulations to discourage the import of agricultural goods. Also, in many European countries, farmers are heavily subsidized, which can be viewed as a form of income redistribution from the modern industrial sector to the inefficient agricultural sector. Notwithstanding the political pressure from the farmers, it is possible that protectionism is a better economic policy than free trade. In this paper we try to justify this claim. Under the framework of linear technology and perfect competition, we suppose that the government of a small open economy is using a nonlinear (progressive) tax system to redistribute income from skilled workers to unskilled workers. Under imperfect information it may be Pareto optimal to impose a tariff on the commodity which is unskilled-labour intensive. The reason is that the tariff raises the domestic price of the imported goods and hence causes the wages of the unskilled worker to go up. This reduces the tax burden on the skilled labour and at the same time relaxes the incentive constraint on the skilled workers to mimic the unskilled workers. The distortion resulting from the tariff, on the other hand, is of second order magnitude. The overall welfare effect of the tariff is therefore positive.

In Section 2 we lay out the setup of the model. In Section 3 we investigate the government’s optimization problem and show that a positive tariff can be welfare improving. Section 4 discusses the effect of the tariff if consumption taxes and production subsidies are available and section 5 concludes. The model is an extension of Stiglitz (1982) and a modification of Naito (1999).

\textsuperscript{2}See Brander and Spencer (1984).
2 Setup of the Model

We consider a small open economy with two types of consumers, skilled workers $s$ and unskilled workers $u$. We assume that the population of each type is the same so that we can normalize them to one. There are two goods. Good 1 is an unskilled-labour intensive good and good 2 is a skilled-labour intensive good. We first describe the various agents in this economy and then impose the equilibrium conditions.

2.1 The Consumers

In addition to the usual regularity conditions, we assume that the utility function is weakly separable between the consumption goods and labour supply. That is,
\[ u^i = V^i(U(c^i_1, c^i_2), L^i) \]
for \( i = s, u \), where \( c^i_1 \) and \( c^i_2 \) are the quantities of good 1 and good 2 consumed by the type \( i \) workers, and \( L^i \) is the labour supply. Suppose that income \( x \) is given. Separability implies that the choice between the two goods is independent of \( L^i \). We can define a sub-indirect utility function in terms of the prices and income:
\[ v(p + \tau, x) = \max \{ U(c^i_1, c^i_2) | (p + \tau) c^i_1 + c^i_2 \leq x \} \]
where \( p \) is the international price of good 1 and \( \tau \) is the imposed tariff. The price of good 2 is normalized to 1. Using Roy’s identity, we have
\[ \frac{\partial v(p + \tau, x)}{\partial p} = -D^1(p + \tau, x) \frac{\partial v(p + \tau, x)}{\partial x} \]  
(1)

where \( D^1 \) is the ordinary demand function for good 1. Also, putting the sub-indirect utility function \( v \) into the utility function gives us the conditional indirect utility function:
\[ u^i = V^i(v(p + \tau, x), L) \]
for \( i = s, u \).

2.2 The Firms

There are two firms in the economy, each producing one good. Technology is represented by the following two concave and linearly homogeneous production functions:
\[ y_1 = F^1(l^s_1, l^u_1), \quad y_2 = F^2(l^s_2, l^u_2) \]
where \( y_1 \), \( l^s_1 \) and \( l^u_1 \) are the output, skilled labour input, and unskilled labour input respectively of firm \( 1 \). Because of the linear homogeneity assumption, the cost functions are separable in wages and output. Let the unit cost functions be \( C_i(w_s, w_u) \), \( i = 1, 2 \). If we assume perfect competition, then
\[ C_1(w_s, w_u) = p + \tau, \quad C_2(w_s, w_u) = 1. \]  
(3)
By Shephard’s Lemma, the labour demand in industry $k$ are
\[
I^s_k = y_k \frac{\partial C_k(w_s, w_u)}{\partial w_s}, \quad I^u_k = y_k \frac{\partial C_k(w_s, w_u)}{\partial w_u}, \quad k = 1, 2. \tag{4}
\]
Given $p + \tau$, $w_s$ and $w_u$ can be determined using Equation (3):
\[
w_s = w_s(p + \tau), \quad w_u = w_u(p + \tau), \quad \frac{w_u}{w_s} = \Omega = \Omega(p + \tau).
\]
Since we assume that good 1 is unskilled-labour intensive and good 2 is skilled-labour intensive, by the Stolper-Samuelson theorem the effect of changes in price good 1 on the wages are
\[
w'_s(p + \tau) < 0, \quad w'_u(p + \tau) > 0, \quad \Omega'(p + \tau) > 0. \tag{5}
\]
We also assume that in equilibrium both goods are produced and the country imports good 1 and exports good 2. Also, the economy is small enough so that small changes in consumption and production do not affect world prices.

2.3 The Government

We assume that the government can only observe the workers’ income but not their types. Let $R^i$, $i = s, u$ be the total labour income. Then the amount of income tax or subsidy is a function of $R^i$, i.e., $T_i = T(R^i)$. The net income of the consumers are therefore $x^i = R^i - T_i$, $i = s, u$. Given the function $T$, the consumers choose their labour supply to maximize utility. Equivalently, the government can provide an “income menu” $\{(R^s, x^s), (R^u, x^u)\}$. Note that with total labour income $R^i$, the labour supply is $R^i/w_i$ for type $i$ worker, where $i, j = s, u$. To prevent one type of workers to mimic the other type, the income menu has to satisfy the following two incentive constraints:
\[
V^s(v(p + \tau, x^s), R^s/w_s) \geq V^s(v(p + \tau, x^u), R^u/w_s), \tag{6}
\]
\[
V^u(v(p + \tau, x^u), R^u/w_u) \geq V^u(v(p + \tau, x^s), R^s/w_u). \tag{7}
\]
Inequality (6) ensures that a skilled worker will not pose as an unskilled worker by choosing $L = R^u/w_s$ and getting net income $x^u$. Inequality (7) is similar for the unskilled workers.

The government budget constraint is
\[
T_s + T_u + \tau(c^s_1 + c^u_1 - y_1) \geq 0.
\]
$T_s + T_u$ are the income tax revenues (or subsidies if negative), while the third term is the revenue from the tariff. Since $T_i = R^i - x^i = w_i L^i - x^i$, the above constraint can be written as
\[
w_s L^s + w_u L^u + \tau(c^s_1 + c^u_1 - y_1) \geq x^s + x^u. \tag{8}
\]
2.4 Equilibrium

Assuming labour is free to move between the two industries, equilibrium conditions in the labour markets are

\[ L^s = l^s_1 + l^s_2, \quad L^u = l^u_1 + l^u_2. \]  

(9)

Since technology is convex and factor intensity is different in the two industries, the production possibility set is strictly convex. For given output prices and fixed total labour supply, the outputs are uniquely determined. Formally, using (4) and (9), we can solve for \( y_1 \) and \( y_2 \) in terms of \( p + \tau, L^s \) and \( L^u \):

\[ y_1 = Y^1(p + \tau, L^s, L^u), \]

\[ y_2 = Y^2(p + \tau, L^s, L^u). \]

Since the production possibility set is strictly convex,

\[ \frac{\partial Y^1}{\partial (p + \tau)} > 0, \]

and

\[ \frac{\partial Y^2}{\partial (p + \tau)} < 0. \]

Also, by the Rybczynski theorem, \(^3\)

\[ \frac{\partial Y^1}{\partial L^s} < 0, \quad \frac{\partial Y^1}{\partial L^u} > 0, \]

\[ \frac{\partial Y^2}{\partial L^s} > 0, \quad \frac{\partial Y^2}{\partial L^u} < 0. \]

The balance-of-trade condition is

\[ y_2 - c^s_2 - c^u_2 = p(c^s_1 + c^u_1 - y_1). \]  

(10)

That is, the value of exports (good 2) is equal to the value of imports. Substituting \( c^s_1 + c^u_1 - y_1 \) in (10) into (8), the government budget constraint becomes

\[ w_s L^s + w_u L^u + (y_2 - c^s_2 - c^u_2) \frac{\tau}{p} \geq x^s + x^u. \]

This can be rewritten as

\[ w_s L^s + w_u L^u + [Y^2(p + \tau, L^s, L^u) - D^2(p + \tau, x^s) - D^2(p + \tau, x^u)] \frac{\tau}{p} \geq x^s + x^u \]  

(11)

where \( D^2(p + \tau, x^i) \) is the ordinary demand function of the type \( i \) consumer for good 2.


5
3 Effect of the Tariff on Welfare

Using the setup in Section 2, we now investigate the effect of a tariff on social welfare. It is natural to assume that in equilibrium the wages of the skilled workers are higher than that of the unskilled workers. The government is using a nonlinear income tax system (the income menu) to redistribute income from the skilled workers to be unskilled workers. Since wages are determined by the international prices of goods \( p \) and the tariff rate \( \tau \), controlling the total labour income \( R \) is equivalent to controlling labour supply \( L \). In the following optimization problem, we use \( \{L^s, L^u, x^s, x^u\} \) as the control variable:

\[
\max V^s(v(p + \tau, x^s), L^s)
\]

subject to

\[
V^u(v(p + \tau, x^u), L^u) \geq V^s(v(p + \tau, x^u), \Omega(p + \tau)L^u), \tag{13}
\]

\[
V^u(v(p + \tau, x^u), L^u) \geq V^a(v(p + \tau, x^s), \frac{L^s}{\Omega(p + \tau)}), \tag{14}
\]

\[
w^s L^s + w^u L^u + [Y^2(p + \tau, L^s, L^u) - D^2(p + \tau, x^s) - D^2(p + \tau, x^u)] \frac{\tau}{p} \geq x^s + x^u. \tag{15}
\]

Given any utility level \( u^* \) of the unskilled workers in (12), the government maximized the utility of the skilled workers. Hence the solution is second best under the nonlinear tax system. (13) and (14) are the incentive compatibility constraints in (6) and (7), where we have replaced \( R^j/w_i \) with \( w_j L_j/w_i \) and used the definition \( \Omega = w_u/w_s \). (15) is the government budget constraint from (11). Before writing down the first order conditions, we define the following quantities for notational convenience:

\[
\frac{\partial V^i(v(p + \tau, x^i), L^i)}{\partial v(p + \tau, x^i)} \equiv V^{ii}_1
\]

\[
\frac{\partial V^i(v(p + \tau, x^i), L^i)}{\partial L^i} \equiv V^{ii}_2
\]

\[
\frac{\partial V^i(v(p + \tau, x^i), \frac{w_j L_j}{w_i})}{\partial v(p + \tau, x^i)} \equiv V^{ij}_1, \quad i \neq j
\]

\[
\frac{\partial V^i(v(p + \tau, x^i), \frac{w_j L_j}{w_i})}{\partial \frac{w_j L_j}{w_i}} \equiv V^{ij}_2, \quad i \neq j
\]

\[
\frac{\partial v(p + \tau, x^i)}{\partial (p + \tau)} \equiv v_p(p + \tau, x^i)
\]

\[
\frac{\partial v(p + \tau, x^i)}{\partial x^i} \equiv v_i
\]
\[ \frac{\partial Y^2(p + \tau, L^s, L^u)}{\partial L^i} \equiv Y^2_i \]
\[ \frac{\partial D^2(p + \tau, x^i)}{\partial x^i} \equiv D^2_i \]

for \( i, j = s, u \).

Assuming interior solutions, that is, \( L^s, L^u, x^s, x^u > 0 \). The first order conditions are:

\[
L^s : V_2^{ss} + \lambda_1 V_2^{ss} - \frac{\lambda_2 V_2^{us}}{\Omega} + \lambda_3 (w_s + \frac{\tau Y^2_s}{p}) = 0, \\
L^u : \mu V_2^{uu} - \lambda_1 V_2^{su} \Omega + \lambda_2 V_2^{uu} + \lambda_3 (w_u + \frac{\tau Y^2_u}{p}) = 0, \\
x^s : V_1^{ss} v_s + \lambda_1 V_1^{ss} v_s - \lambda_2 V_1^{us} v_s - \lambda_3 \left( \frac{\tau D^2_s}{p} + 1 \right) = 0, \\
x^u : \mu V_1^{uu} v_u - \lambda_1 V_1^{su} v_u + \lambda_2 V_1^{uu} v_u - \lambda_3 \left( \frac{\tau D^2_u}{p} + 1 \right) = 0.
\]

where \( \mu, \lambda_1, \lambda_2, \) and \( \lambda_3 \) are the Lagrange multipliers of the constraints in (12), (13), (14) and (15) respectively.

For a given international price \( p \), the solution of the optimization problem depends on \( \tau \). Hence we can define the optimized objective function as

\[ W(\tau) = V^*(v(p + \tau, x^s(\tau)), L^s(\tau)). \]

Suppose the country is initially under free trade. We want to investigate whether it is Pareto improving to impose a tariff on good 1. In other words, we want to determine the sign of \( dW/d\tau \) at \( \tau = 0 \). Let \( \mathcal{L} \) be the Lagrangian of the problem. Then by the envelope theorem, we have, at \( \tau = 0 \):

\[
dW/d\tau = d\mathcal{L}/d\tau \\
= V_1^{ss} v_p(p, x^s) + \mu V_1^{uu} v_p(p, x^u) \\
+ \lambda_1 [V_1^{ss} v_p(p, x^s) - V_1^{ss} v_p(p, x^u) - V_2^{su} \Omega'(p) L^u] \\
+ \lambda_2 [V_1^{uu} v_p(p, x^u) - V_1^{us} v_p(p, x^s) - V_2^{su} \Omega'(p)/\Omega^2] \\
+ \lambda_3 \{ w'_s(p) L^s + w'_u(p) L^u + \frac{1}{p} [Y^2(p, L^s, L^u) - D^2(p, x^s) - D^2(p, x^u)] \}.
\]
Using Roy’s identity in (1), we have
\[
\frac{dW}{d\tau} = -D^1(p, x^s)[V_1^{ss}v_s + \lambda_1 V_1^{ss}v_s - \lambda_2 V_1^{us}v_s] \\
- D^1(p, x^u)[\mu V_1^{su}v_u - \lambda_1 V_1^{su}v_u + \lambda_2 V_1^{uu}v_u] \\
- \lambda_1 V_2^{su}\Omega'(p)L^u + \lambda_2 V_2^{us}L_s\frac{\Omega'(p)}{\Omega^2} \\
+ \lambda_3\{w_s'(p)L^s + w_u'(p)L^u \\
+ \frac{1}{p}[Y^2(p, L^s, L^u) - D^2(p, x^s) - D^2(p, x^u)]\}. \tag{16}
\]

Using the first order condition for \(x^s\) with \(\tau = 0\), the expression inside the first square bracket is equal to \(\lambda_3\). Similarly, the first order condition for \(x^u\) implies that the expression inside the second square bracket is also \(\lambda_3\). Now (16) becomes
\[
\frac{dW}{d\tau} = -\lambda_1 V_2^{su}\Omega'(p)L^u + \lambda_2 V_2^{us}L_s\frac{\Omega'(p)}{\Omega^2} \\
+ \lambda_3\{w_s'(p)L^s + w_u'(p)L^u \\
+ \frac{1}{p}[Y^2(p, L^s, L^u) - D^2(p, x^s) - D^2(p, x^u)]\}. \tag{17}
\]

Now we want to show that \(w_s'(p)L^s + w_u'(p)L^u = y_1\). Perfect competition implies that profits are zero. In the other words,
\[
r(p, L^s, L^u) = w_s(p)L^s + w_u(p)L^u
\]
where \(r(p, L^s, L^u)\) is the revenue or GDP function. For fixed factor supplies \(L^s\) and \(L^u\), we can apply Hotelling’s Lemma to get
\[
y_1 = \frac{\partial r(p, L^s, L^u)}{\partial p} = w_s'(p)L^s + w_u'(p)L^u. \tag{18}
\]

Using (18), the expression inside the braces in (17) becomes
\[
-\lambda_1 V_2^{su}\Omega'(p)L^u + \lambda_2 V_2^{us}L_s\frac{\Omega'(p)}{\Omega^2},
\]
which is zero by the balance of trade equilibrium in (10). Therefore (17) becomes
\[
\frac{dW}{d\tau} = -\lambda_1 V_2^{su}\Omega'(p)L^u + \lambda_2 V_2^{us}L_s\frac{\Omega'(p)}{\Omega^2}. \tag{19}
\]

Intuitively, \(\lambda_1\) and \(\lambda_2\) are the shadow prices of relaxing the incentive compatibility constraints of the skilled workers and the unskilled workers respectively. The terms

\[\text{See Dixit and Norman (1980), p.30-43.}\]
$V_{su}^u \Omega'(p) L^u$ and $V_{us}^u L^u \Omega'(p)$ measure how much these constraints are relaxed by the change in wage ratio $\Omega$ via a small change in the tariff at the zero level.

The government is using a non-linear tax system to redistribute income. If the tax rates are low at the optimum, then each type of workers has no incentive to mimic the other type. In this case the constraints are nonbinding so that $\lambda_1 = \lambda_2 = 0$. Hence $dW/d\tau = 0$ and we obtain the classical result that for a small open economy that the optimal tariff is zero.

A more interesting case is that the government sets the utility level $u^*$ of the unskilled workers high enough such that the constraint in (13) becomes binding. In this case the skilled workers have the incentive to mimic the unskilled workers but not the other way round, so that $\lambda_1 > 0$ and $\lambda_2 = 0$. Since $V_{su}^u < 0$ and $\Omega'(p) > 0$, we have $dW/d\tau > 0$. Therefore at the zero tariff level, the government has the incentive to impose a tariff on good 1. Under free trade, the government is using a progressive income tax system to redistribute income from the skilled labour to the unskilled labour. When the government imposes a tariff, the domestic price of the imported good goes up. Since the production of this good at home is unskilled-labour intensive, it raises the wages of the unskilled labour and lowers that of the skilled labour. At a higher wage, the unskilled labour can work more while maintaining same level of utility. As a consequence, their higher labour income lessens the tax burden on the skilled labour. This has a direct effect on relaxing the incentive compatibility constraint of the skilled workers, while the distortion created by the tariff are negligible. The overall result is a Pareto improvement. Our result therefore differs from the classical trade theory.

### 4 Tariff, consumption tax, and production subsidy

In this paper we consider the effect of a tariff in the absence of any consumption tax and production subsidy. It is well known that if these policy instruments are available, a tariff on a particular good is equivalent to a combination of a consumption tax and a production subsidy on the same good at equal rates.\(^5\) Therefore an optimal tariff on good 1 means that it is also optimal to impose a consumption tax and a production subsidy at the same rate if there is no tariff. Since the latter condition is very unlikely to be satisfied, one can argue that domestic taxes and subsidies are better policy instrument than tariffs. Consumption taxes, however, are political unpopular. In many countries farm products are exempted from consumption taxes. Therefore under such situations a tariff can be used in combination with a production subsidy to achieve an optimal tax policy. And in this paper we have shown that a tariff is an additional convenient policy instrument for a progressive-tax regime.

fact, combinations of subsidies and tariffs are commonly used by many European
governments in the agricultural sector.

5 Conclusion

This paper uses a standard two by two general equilibrium model to investigate the
effect of a tariff in a small open economy under the assumptions of linear technol-
ogy and perfect competition. The government is using progressive income taxes to
redistribute income. Under the condition of imperfect information and with a high
enough tax level, a tariff may be Pareto improving. This result differs from the
classical trade theory, which advocates free trade. In explaining the trade policies
of many advanced and developing economies, this paper is complimentary to the
strategic trade literature, which is more applicable to the modern industrial sector.
In the presence of production subsidies, a tariff can be used as an extra instrument
to achieve an optimal tax policy.

References

Competition’ in: Gene M. Grossman, ed., Imperfect Competition and Interna-

Dixit, Avinash (1985) ‘Tax Policy in Open Economies’ in: Alan J. Auerbach and
Martin Feldstein, ed., Handbook of Public Economics, (North-Holland, Ams-
tterdam) 313-374.

Dixit, A. K. and V. Norman, (1980) Theory of International Trade, (Cambridge Uni-

Competition in the Presence of Oligopoly and Economies of Scale’, in: Gene
M. Grossman, ed., Imperfect Competition and International Trade, (The MIT

Naito, Hisahiro (1999) ‘Re-examination of Uniform Commodity Taxes under a Non-
linear Income Tax System and Its Implication for Production Efficiency’, Journal of
Public Economics,71, 165-188.

Economics, 17, 213-240.