Novel adaptive neural control design for nonlinear MIMO time-delay systems

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A B S T R A C T

In this paper, we address the problem of adaptive neural control for a class of multi-input multi-output (MIMO) nonlinear time-delay systems in block-triangular form. Based on a neural network (NN) online approximation model, a novel adaptive neural controller is obtained by constructing a novel quadratic-type Lyapunov–Krasovskii functional, which not only efficiently avoids the controller singularity, but also relaxes the restriction on unknown virtual control coefficients. The merit of the suggested controller design scheme is that the number of online adapted parameters is independent of the number of nodes of the neural networks, which reduces the number of the online adaptive learning laws considerably. The proposed controller guarantees that all closed-loop signals remain bounded, while the output tracking error dynamics converges to a neighborhood of the origin. A simulation example is given to illustrate the design procedure and performance of the proposed method.

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1. Introduction


Recently, in view of possible time-delays in practical systems, approximation-based adaptive neural network control has been also addressed in Ge, Hong, and Lee (2003) for nonlinear SISO time-delay systems with constant virtual control coefficients. An adaptive neural controller has been developed to guarantee that all the closed-loop signals remain bounded, meanwhile output tracking is achieved. Such a result, with the use of Nussbaum-type functions, has been extended to the case of unknown virtual control coefficients in Ge et al. (2004). In Ho, Li, and Niu (2005), based on a wavelet neural network online approximation model, a state feedback adaptive controller is proposed by constructing an integral-type Lyapunov–Krasovskii functional. More recently, in Ge and Tee (2007), neural control has been addressed for MIMO nonlinear time-delay systems in block-triangular form. The suggested adaptive neural controller guarantees that the tracking errors converge to a small neighborhood of the origin, and at the same time, all other signals in the closed-loop system are bounded.

Though various approximation-based neural control design methods have been proposed for delay-free systems or delayed systems, there are still some issues which need to be further addressed. When a neural network is used as an approximator, to get the sufficient approximation accuracy, a large number of NN nodes should be adopted. As a result, a great number of adaptation parameters are required to be adapted online simultaneously. This makes the learning time become unacceptably large. In addition, in the above-mentioned adaptive neural control design,
even though an integral-type Lyapunov function has been used to successfully avoid the controller singularity problem, it has the following drawbacks: The upper bounds for unknown virtual control coefficients must be some known functions and real control coefficients should be independent of some specified state variables and control inputs. This may limit the applicability of the approach to certain practical systems.

The above observation motivates the research in this paper. A novel adaptive neural control design procedure is proposed for MIMO nonlinear time-delay systems in block-triangular form. It is shown that the proposed adaptive control scheme not only avoids the controller singularity problem, but also reduces the number of adaptation parameters considerably. As a matter of fact, to control an m input system, there are only m adaptive laws. In addition, by adopting a new Lyapunov functional, the existing restrictions on the virtual or real control coefficients are removed. The proposed control scheme guarantees the boundedness of all the signals in the closed-loop system, at the same time output tracking is achieved.

2. Problem formulation and preliminaries

Consider the following MIMO nonlinear time-delay system with block-triangular structure.

\[
\begin{align*}
\dot{x}_{ij} &= f_{ij}(\bar{x}_{ij}) + g_{ij}(\bar{x}_{ij})x_{ij} + h_{ij}(\bar{x}_{ij}), \\
x_{ij,m} &= f_{ij,m}(X, \bar{u}_{i-1}) + g_{ij,m}(X, \bar{u}_{i-1})u_i + h_{i,j,m}(X_i), \quad (i, j) \in \Omega,
\end{align*}
\]

for \( j = 1, 2, \ldots, n, i_j = 1, 2, \ldots, m_j - 1 \), where \( \bar{x}_{ij} = [x_{ij,1}, \ldots, x_{ij,j}]^T \in \mathbb{R}^j \) is the state vector for the first \( i \) differential equations of the \( j \)th subsystem, \( y = [y_1, y_2, \ldots, y_i]^T \in \mathbb{R}^j \) is the output, \( \bar{u}_i = [u_1, \ldots, u_i]^T \) are the inputs for the first \( i_j \) subsystems, \( f_{ij}(), g_{ij}() \) and \( h_{ij}() \) are unknown smooth nonlinear functions, \( X = [x_1^T, \ldots, x_i^T]^T \) with \( x_i = [x_{ij,1}, \ldots, x_{ij,m_j}]^T \), \( x_{ij,t} = x_{ij,t} - t - \tau_{ij} \) denotes the delayed state, and \( \bar{x}_{ij} \) and \( X_i \) are defined as

\[
\bar{x}_{ij} = [x_{ij,1}, \ldots, x_{ij,j}]^T, \\
X_i = [x_{1,1}, \ldots, x_{i,1}, \ldots, x_{n,1}, \ldots, x_{n,m_j}]^T
\]

and \( \tau_{ij} \) is the unknown constant time-delay. For \( t \in [-\tau_{ij}, 0] \), let \( x_{ij,t}(t) = \phi_{ij}(t) \), which are assumed to be bounded.

Remark 1. In plant (1), each control gain function \( g_{ij,m}(.) \) contains all state variables and the inputs of the previous subsystems. This is different from the case in Ge and Tee (2007), where \( g_{ij,m} \) do not contain the control input \( u_i (1 \leq l < j) \) and also \( x_{k,m}(k = j + 1, \ldots, n) \). Obviously, the system considered here is more general.

As usually done, the following assumptions are made for system (1).

Assumption 1. The desired trajectories \( y_{d_j}, j = 1, 2, \ldots, n \), and their time derivatives up to the 9th order, are continuous and bounded.

Assumption 2. The signs of \( g_{ij,m}(.) \) are known and there exist some unknown constant \( g_0 \) and unknown smooth functions \( \hat{g}_{ij,j}(.) \) such that \( 0 < g_0 \leq |g_{ij,j}(.)| \leq \hat{g}_{ij,j}(.) < \infty \).

Remark 2. Apparently, Assumption 2 implies that \( g_{ij,j}(.) \) is strictly positive or negative. Without loss of generality, we further assume \( \hat{g}_{ij,j}(.) > g_0 > 0 \). In addition, notice that in Assumption 2 the upper bounds \( g_{ij,j}(.) \) are unknown. Therefore, Assumption 2 in this paper is less restrictive than that in Ho et al. (2005) and Ge and Tee (2007), where \( \hat{g}_{ij,j}(.) \) are required to be known for constructing control laws.

Assumption 3. There exist positive functions \( Q_{lj}(x_{lj}) \) for \( l = 1, 2, \ldots, i_j \) such that \( |h_{lj}(\bar{x}_{lj})| \leq \sum_{l=1}^{i_j} Q_{lj}(x_{lj}) \).

In this paper, the following radial basis function (RBF) NNs will be used as an approximator to approximate an unknown continuous function. As pointed out in Sanner and Slotine (1992), for a given \( \varepsilon > 0 \) and any continuous function \( f(Z) \) defined on \( \Omega < \mathbb{R} \), there exists an NN \( W_i(S(Z)) \) such that

\[
f(Z) = W_i(S(Z)) + \delta(Z), \quad |\delta(Z)| \leq \varepsilon,
\]

where \( Z \in \Omega \subset \mathbb{R} \) is the input vector, \( W = [w_1, w_2, \ldots, w_n]^T \) is the weight vector, \( l > 1 \) is the number of the NN nodes and \( S(Z) = [s_1(Z), \ldots, s_l(Z)]^T \) is defined by

\[
s_i = \exp \left[ -\frac{(Z - \mu_i)^T(Z - \mu_i)}{\phi_i^2} \right], \quad i = 1, 2, \ldots, l
\]

with \( \mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{in}]^T \) the center of the receptive field and \( \phi_i \) the width of the Gaussian function.

3. Adaptive NN control design

This section is devoted to developing a novel adaptive NN control design procedure. The design procedure for the \( j \)th subsystem is composed of \( m_j \) design steps. In each step, the radial basis function NN \( W_i(S(Z)) \) will be used to approximate the unknown nonlinear function \( f_{ij} \). Thus, define an unknown constant as

\[
\theta_j = \frac{1}{g_0} \max \{ \|W_{ij} \|^2 : 1 \leq i \leq m_j \}
\]

where the constant \( g_0 \) is defined as in Assumption 2, function \( f_{ij} \) and vector \( Z_{ij} \) will be specified in each step. Furthermore, for \( j = 1, \ldots, n \) and \( i_j = 1, \ldots, m_j - 1 \), the virtual control laws \( \alpha_{ij}(.) \) are chosen as follows:

\[
\alpha_{ij}(t) = -\left( k_{ij} + \frac{1}{2} \right) z_{ij} - \frac{1}{2} \hat{\theta}_j z_{ij} S(Z_{ij}) S(Z_{ij}),
\]

where \( k_{ij} > 0 \) and \( \alpha_{ij} > 0 \) are design parameters, \( \hat{\theta}_j \) is the estimation of the unknown constant \( \theta_j \), \( S(.) \) is the basis function vector, and the variables \( Z_{ij} \) are defined by

\[
z_{ij} = x_{ij} - y_{d_j}, \quad z_{ij} = x_{ij} - y_{d_j}
\]

for \( j = 1, \ldots, n \) and \( i_j = 2, \ldots, m_j \). The adaptive laws \( \hat{\theta}_j \) for \( j = 1, \ldots, n \) are given by

\[
\hat{\theta}_j(t) = \sum_{l=1}^{m_j} \frac{r_j}{2g_{ij,l}^2} z_{ij}^l S(Z_{ij}) S(Z_{ij}) - b_{ij}
\]

where \( r_j > 0 \) and \( b_{ij} > 0 \) are design parameters.

Step 1. For the first differential equation of the \( j \)th subsystem, consider the Lyapunov function as follows:

\[
V_{z_{ij}} = \frac{1}{2} z_{ij}^l + \frac{g_0}{2r_{ij}} \frac{\hat{\theta}_j^2}{2}.
\]

where \( z_{ij} = x_{ij} - y_{d_j}, \hat{\theta}_j = \theta_j - \hat{\theta}_j \) is the estimate of \( \theta_j \). Then, the time derivative of \( V_{z_{ij}} \) is given by
\[ \dot{V}_{z_{1j}} = z_{1j} \left( f_{ij} + g_{ij} \alpha_{ij} - \dot{y}_{ij} + h_{ij} (\dot{x}_{ij}) \right) \]
\[ + z_{1j} \delta_{x_{ij}} z_{2j} - \frac{g_{ij}}{f_{ij}} \hat{\theta}_{ij}. \]  
(6)

With Assumption 3, completing the square gives
\[ z_{1j} h_{ij} (\dot{x}_{ij}) \leq |z_{1j}| Q_{ij}^{l}(x_{ij}) \leq \frac{1}{2} z_{1j}^{2} + \frac{1}{2} \left[ Q_{ij}^{l}(x_{ij}) \right]^{2}. \]

Substituting this inequality into (6) yields
\[ \dot{V}_{z_{1j}} \leq z_{1j} \left( f_{ij} + g_{ij} \alpha_{ij} - \dot{y}_{ij} + \frac{1}{2} z_{2j}^{2} \right) \]
\[ + z_{1j} \delta_{x_{ij}} z_{2j} + \frac{1}{2} \left[ Q_{ij}^{l}(x_{ij}) \right]^{2} - \frac{g_{ij}}{f_{ij}} \hat{\theta}_{ij}. \]  
(7)

To deal with the delay term in (7), consider the Lyapunov–Krasovskii functional as follows: \( V_{\theta_{ij}} = V_{z_{ij}} + V_{\theta_{ij}} \) with
\[ V_{\theta_{ij}} = \int_{t_{ij}}^{t} \frac{1}{2} \left[ \left( Q_{ij}^{k}(x(s)) \right) \right]^{2} ds. \]

Differentiating \( V_{\theta_{ij}} \) and using (7), the inequality below can be obtained easily.
\[ \dot{V}_{\theta_{ij}} \leq z_{1j} \left( f_{ij}(Z_{ij}) + g_{ij} \alpha_{ij} \right) + z_{1j} \delta_{x_{ij}} z_{2j} - \frac{g_{ij}}{f_{ij}} \hat{\theta}_{ij} \]
\[ + \left[ 1 - 2 \tanh^{2} \left( \frac{z_{1j}}{\eta_{ij}} \right) \right] U_{ij}, \]  
(8)

where \( \eta_{ij} = [x_{ij}, y_{ij}, \dot{x}_{ij}]^{T} \), \( U_{ij} = \frac{1}{2} \left[ Q_{ij}^{m}(x) \right]^{2} \) and
\[ f_{ij}(Z_{ij}) = f_{ij} - \dot{y}_{ij} + \frac{1}{2} z_{2j}^{2} \tanh^{2} \left( \frac{z_{1j}}{\eta_{ij}} \right) \]
with \( \eta_{ij} \) being a positive constant. As pointed out by Remark 5 in Ge and Tee (2007), the function \( \frac{1}{2} \tanh^{2} \left( \frac{1}{\eta} \right) \) is well defined at \( z = 0 \) and can be approximated by a neural network. So, the NN \( W_{ij}^{T} S(Z_{ij}) \) is utilized to approximate \( f_{ij} \) such that for given \( \delta_{ij} > 0 \),
\[ f_{ij}(Z_{ij}) = W_{ij}^{T} S(Z_{ij}) + \delta_{ij}(Z_{ij}), \quad |\delta_{ij}| \leq \epsilon_{ij}, \]
where \( \delta_{ij} \) denotes the approximation error. Furthermore, a straightforward calculation shows
\[ z_{1j} \dot{U}_{ij}(Z_{ij}) \leq \frac{1}{2a_{ij}} g_{ij} \delta_{x_{ij}}^{2} \delta_{x_{ij}} S^{T}(S(Z_{ij})) S(Z_{ij}) \]
\[ + \frac{1}{2} \delta_{x_{ij}}^{2} + \frac{1}{2} g_{ij} \delta_{x_{ij}}^{2} + \frac{1}{2} \delta_{x_{ij}}^{2} \delta_{ij} \]  
(9)

where \( a_{ij} > 0 \) is a design parameter. In addition, from (5), it can be verified that for any initial conditions \( \hat{\theta}_{ij}(t_{ij}) \geq 0, \hat{\theta}_{ij}(t) \geq 0 \) for all \( t \geq t_{ij} \). Consequently, it follows that
\[ z_{1j} \delta_{ij} \alpha_{ij} \leq - \frac{g_{ij}}{2a_{ij}} \hat{\theta}_{ij} \delta_{x_{ij}}^{2} S^{T}(S(Z_{ij})) S(Z_{ij}) - \left( k_{ij} + \frac{1}{2} \right) g_{ij} \delta_{x_{ij}}^{2}. \]  
(10)

Thus, substituting (9) and (10) into (8) results in
\[ \dot{V}_{\theta_{ij}} \leq -k_{ij} g_{ij} \delta_{x_{ij}}^{2} + \frac{1}{2} \left( a_{ij}^{2} + \delta_{x_{ij}}^{2} g_{ij}^{2} \right) \]
\[ + \frac{g_{ij}}{f_{ij}} \hat{\theta}_{ij} \left( \frac{f_{ij}}{2a_{ij}} \delta_{x_{ij}}^{2} S^{T}(S(Z_{ij})) S(Z_{ij}) - \hat{\theta}_{ij} \right) \]
\[ + z_{1j} \delta_{ij} z_{2j} + \left[ 1 - 2 \tanh^{2} \left( \frac{z_{1j}}{\eta_{ij}} \right) \right] U_{ij}, \]  
(11)

**Step (i,j) (for \( i_{j} = 2, \ldots, m_{j} - 1 \)).** Consider the following Lyapunov–Krasovskii functional
\[ V_{\theta_{ij}} = \frac{1}{2} z_{ij}^{2} \]

Differentiating \( V_{\theta_{ij}} \) to get that
\[ \dot{V}_{\theta_{ij}} = z_{ij} \left( f_{ij} + g_{ij} \alpha_{ij} \right) + \left[ 1 - 2 \tanh^{2} \left( \frac{z_{ij}}{\eta_{ij}} \right) \right] U_{ij}, \]
(12)

By Assumption 3 and completion of the square, we have
\[ z_{1j} h_{ij} (\dot{x}_{ij}) \leq \sum_{k=1}^{h} |z_{1j}| Q_{ij}^{l}(x_{ij}) \]
\[ \leq \sum_{k=1}^{h} \frac{1}{2} z_{ij}^{2} + \sum_{k=1}^{h} \frac{1}{2} \left[ Q_{ij}^{l}(x_{ij}) \right]^{2}. \]  
(13)

Notice that \( \dot{\alpha}_{ij-1}(Z_{ij}) \) can be expressed as
\[ \dot{\alpha}_{ij-1} = \sum_{k=0}^{i-1} \frac{\partial \alpha_{ij-1}}{\partial x_{ij}} h_{ij}(x_{ij}) \]
\[ + \sum_{k=0}^{i-1} \frac{\partial \alpha_{ij-1}}{\partial y_{ij}} y_{ij} + \frac{\partial \alpha_{ij-1}}{\partial \theta_{ij}}. \]  
(14)

Similar to (13), we have
\[ z_{ij} h_{ij} (\dot{x}_{ij}) \]
\[ \leq \sum_{k=1}^{h} \frac{1}{2} z_{ij}^{2} \left( \left[ \frac{\partial \alpha_{ij-1}}{\partial x_{ij}} \right]^{2} + \sum_{k=1}^{h} \frac{1}{2} \left[ Q_{ij}^{l}(x_{ij}) \right]^{2} \right). \]  
(15)

By utilizing (13)–(15), (12) can be rewritten in the following form.
\[ \dot{V}_{\theta_{ij}} \leq z_{ij} \left( f_{ij} + g_{ij} x_{ij} + 1 \right) + \sum_{k=1}^{h} \frac{1}{2} z_{ij}^{2} \]
\[ - \sum_{k=1}^{i-1} \frac{\partial \alpha_{ij-1}}{\partial x_{ij}} h_{ij}(x_{ij}) + \sum_{k=1}^{i-1} \frac{\partial \alpha_{ij-1}}{\partial y_{ij}} y_{ij} + \frac{\partial \alpha_{ij-1}}{\partial \theta_{ij}} \]
\[ + \sum_{k=1}^{h} \frac{1}{2} \left[ Q_{ij}^{l}(x_{ij}) \right]^{2} + \sum_{k=1}^{h} \frac{1}{2} \left[ Q_{ij}^{l}(x_{ij}) \right]^{2}. \]  
(16)

To compensate the delay terms, define an integral function as follows:
\[ V_{\theta_{ij}} = \sum_{k=1}^{h} \int_{t_{ij}}^{t} \frac{1}{2} \left[ Q_{ij}^{k}(x(s)) \right]^{2} ds \]
\[ + \sum_{k=1}^{h} \int_{t_{ij}}^{t} \frac{1}{2} \left[ Q_{ij}^{k}(x(s)) \right]^{2} ds. \]

Differentiating \( V_{\theta_{ij}} \) yields
\[ \dot{V}_{\theta_{ij}} = U_{ij} - \sum_{k=1}^{h} \frac{1}{2} \left[ Q_{ij}^{l}(x_{ij}) \right]^{2} - \sum_{k=1}^{h} \sum_{k=1}^{h} \frac{1}{2} \left[ Q_{ij}^{l}(x_{ij}) \right]^{2} \]
\[ = z_{ij} \frac{1}{2} \left[ \sum_{k=1}^{h} \left( \frac{z_{ij}}{\eta_{ij}} \right) U_{ij} + \left[ 1 - 2 \tanh^{2} \left( \frac{z_{ij}}{\eta_{ij}} \right) \right] U_{ij} \right]. \]
\[
- \sum_{k=1}^{j} \frac{1}{2} \left[ Q^{jk}_{ij} (x_{T_k}) \right]^2 - \sum_{k=1}^{j-1} \sum_{l=1}^{j} \frac{1}{2} \left[ Q^{lk}_{ij} (x_{T_l}) \right]^2 \]

where \( U_{ij} = \sum_{k=1}^{j} \frac{1}{2} \left[ Q^{jk}_{ij} (x_{T_k}) \right]^2 + \sum_{k=1}^{j-1} \sum_{l=1}^{j} \frac{1}{2} \left[ Q^{lk}_{ij} (x_{T_l}) \right]^2 \).

It is clearly seen that by adding (17) to (16) the delay terms are cancelled out. Hence, by utilizing (16) and (17), it can be proved that the derivative of the Lyapunov–Krasovskii functional \( V_{j,ij} = V_{j,ij} + V_{ij} \) satisfies
\[
\dot{V}_{j,ij} \leq z_{ij} \left( \varphi_{ij} - \frac{\partial \varphi_{ij-1}}{\partial \theta_j} \hat{\theta}_j \right) + \left[ 1 - 2 \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) \right] U_{ij}(x) + z_{ij} \left( \tilde{f}_{ij} + g_i \varphi_{ij} + g_j z_{ij} \right) z_{ij+1},
\]
where the equality \( U_{ij} = z_{ij} \frac{2}{\partial z_{ij}} \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) U_{ij}(x) + \left[ 1 - 2 \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) \right] U_{ij}(x) \) is used, and
\[
\tilde{f}_{ij}(Z_{ij}) = f_{ij} + z_{ij-1} g_i \varphi_{ij} + \frac{1}{2} \sum_{k=1}^{j-1} \frac{1}{2} z_{ij} \left( \frac{2}{\partial z_{ij}} \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) U_{ij}(x) + \left[ 1 - 2 \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) \right] U_{ij}(x) \right)
\]

\[
- \sum_{k=1}^{j-1} \frac{1}{2} \left[ \frac{2}{\partial z_{ij}} \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) U_{ij}(x) + \left[ 1 - 2 \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) \right] U_{ij}(x) \right) \]

\[
+ \frac{1}{2} \sum_{k=1}^{j-1} \sum_{l=1}^{j} \frac{1}{2} \left[ \frac{2}{\partial z_{ij}} \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) U_{ij}(x) + \left[ 1 - 2 \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) \right] U_{ij}(x) \right) \]

\[
Z_{ij} = [\tilde{f}_{ij}, \varphi_{ij}, \ldots, \varphi_{ij}, \hat{\theta}_j]^T.
\]

\[
U_{ij} = \frac{1}{2} \left[ \frac{2}{\partial z_{ij}} \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) U_{ij}(x) + \left[ 1 - 2 \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) \right] U_{ij}(x) \right) \]

**Remark 3.** From (5), \( \hat{\theta}_j \) is evidently a function of the whole state variable \( x_i \). Hence, the term \( \frac{\partial \varphi_{ij-1}}{\partial \theta_j} \) cannot be dealt with as done in Ge and Tee (2007), where it is treated as a part of \( \tilde{f}_{ij} \). So, this makes the control law design more difficult. To overcome this difficulty, we introduce a function \( \psi_{ij} \) to compensate the term \( \frac{\partial \varphi_{ij-1}}{\partial \theta_j} \), which will be specified later.

In what follows, the NN \( W_{ij}^T S(Z_{ij}) \) is used to approximate the unknown \( \tilde{f}_{ij} \), such that for given \( \delta_{ij} > 0 \), the following holds.
\[
\tilde{f}_{ij} = W_{ij}^T S(Z_{ij}) + \delta_{ij} \varphi_{ij}, \quad |\delta_{ij}| \leq \delta_{ij}
\]

where \( \delta_{ij} \) denotes the approximation error. Then, by following a similar line used in the procedure from (9) to (10), we have
\[
\dot{V}_{j,ij} \leq -k_{ij} g_0 z_{ij} + \frac{1}{2} \left( a_{ij}^2 + e_{ij}^2 \right) \theta_j^{-1}
\]

\[
+ \frac{g_0}{r_j} \varphi_{ij} \left( \frac{2}{\partial z_{ij}} \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) U_{ij}(x) + \left[ 1 - 2 \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) \right] U_{ij}(x) \right) + z_{ij} \left( \varphi_{ij} - \frac{\partial \varphi_{ij-1}}{\partial \theta_j} \right) + g_i z_{ij} z_{ij+1}.
\]

**Step (j, m, j).** This is the last step for the \( j \)th subsystem to construct the real control law \( u_i \). Consider the following Lyapunov–Krasovskii functional.
\[
V_{j,m} = \frac{1}{2} z_{ij}^2 + V_{ij},
\]

with \( V_{ij} = \sum_{j=1}^{m} \sum_{k=1}^{m} \frac{1}{2} \left[ Q^{jk}_{ij} (x(x(s))) \right]^2 + \sum_{k=1}^{m-1} \sum_{l=1}^{k} \left[ Q^{lk}_{ij} (x(x(s))) \right]^2 \) and \( z_{ij} = x_{ij} - \alpha_{j,m,ij} \). Then, taking (18) with \( i_j = m_j \) into account results in that
\[
\dot{V}_{j,m} \leq \left[ 1 - 2 \tanh^2 \left( \frac{z_{m_j}}{\eta_{m_j}} \right) \right] U_{m_j}(x) + z_{m_j} \left( \varphi_{m_j} - \frac{\partial \varphi_{m_j-1}}{\partial \theta_j} \right) + z_{m_j} (\tilde{f}_{m_j} + g_i \varphi_{m_j} + g_j z_{m_j} z_{m_j+1}),
\]

where \( \tilde{f}_{m_j}(Z_{m_j}) \) can be defined by (19) with \( i_j = m_j \) and \( Z_{m_j} = [X, y_{d_j}, \ldots, y_{d_m}, \hat{\theta}_j, \ldots, \hat{\theta}_j]^T \). The NN \( W_{m_j}^T S(Z_{m_j}) \) is employed to approximate \( \tilde{f}_{m_j} \) such that for given \( \delta_{m_j} > 0 \), the following expression holds.
\[
\tilde{f}_{m_j} = W_{m_j}^T S(Z_{m_j}) + \delta_{m_j} \varphi_{m_j}, \quad |\delta_{m_j}| \leq \delta_{m_j}.
\]

At the present step, choose the control law \( u_i \) as
\[
u = \frac{1}{2a_{i,m}} \left( z_{m_j} \right) \theta_j S(Z_{m_j}) - \left( k_{m_j} + \frac{1}{2} \right) z_{m_j},
\]

Again repeating the procedure from (9) to (10) gives
\[
\dot{V}_{j,m} \leq -k_{m_j} g_0 z_{m_j} + \frac{1}{2} \left( a_{ij}^2 + e_{ij}^2 \right) \theta_j^{-1}
\]

\[
+ \frac{g_0}{r_j} \varphi_{ij} \left( \frac{2}{\partial z_{ij}} \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) U_{ij}(x) + \left[ 1 - 2 \tanh^2 \left( \frac{z_{ij}}{\eta_{ij}} \right) \right] U_{ij}(x) \right) + z_{m_j} \left( \varphi_{m_j} - \frac{\partial \varphi_{m_j-1}}{\partial \theta_j} \right).
\]

Let \( V = \sum_{j=1}^{m} \sum_{k=1}^{m} \dot{V}_{j,k} \). Then, (11), (18) and (23) imply that
\[
\dot{V} \leq - \sum_{j=1}^{m} \sum_{k=1}^{m} \frac{1}{2} \left( a_{ij}^2 + e_{ij}^2 \right) \theta_j^{-1}
\]

\[
+ \frac{g_0}{r_j} \sum_{k=1}^{m} \sum_{j=1}^{m} \left( z_{m_j} \right) S(Z_{m_j}) - \hat{\theta}_j
\]

\[
+ \sum_{j=1}^{m} \sum_{k=1}^{m} \left[ 1 - 2 \tanh^2 \left( \frac{z_{m_j}}{\eta_{ij}} \right) \right] U_{ij}(x) + \sum_{j=1}^{m} \sum_{k=1}^{m} \left( \varphi_{i,k} - \frac{\partial \varphi_{i,k-1}}{\partial \theta_j} \right).
\]

So far, we have completed the control law design.

**Remark 4.** In the existing neural adaptive control design approaches, each weight vector is just the estimated vector. Therefore, the number of the adaption parameters depends on the number of the NN nodes. Consequently, if a system contains a large number of unknown nonlinear function, or more NN nodes are used to improve the approximation precision, there will be a large number of adaption parameters that need to be updated.
online simultaneously. This makes the learning time become unacceptably large. Instead of estimating each element in the weight vector W, we estimate the norm of all the weight vectors, so only one adaptation learning law is required to control each subsystem.

4. Stability Analysis

In this section, the boundedness of all the signals in the closed-loop system will be proved. The main result is summarized in the following theorem.

**Theorem.** Consider system (1) satisfying Assumptions 1–3. Suppose that for 1 ≤ j ≤ m, 1 ≤ i ≤ m, the packaged unknown functions \( \hat{f}_{j,i} \) can be approximated by neural network in the sense that the approximating error \( \| e \| \) is acceptably large. In this paper, inspired by the idea in Yang, Feng, and Ren (2004), instead of estimating each element in the weight vector \( \theta \), we choose Lyapunov functional as

\[
V(x) = \sum_{j=1}^{n} \frac{1}{2} x_j^2 + \sum_{i=1}^{m} \frac{1}{2} \sum_{j=1}^{n} \frac{1}{2} x_j^2
\]

where \( V \) is a new Lyapunov function. Then, combining the specified control gains, system (1) is then stable. It means that all the closed-loop trajectories remain bounded.

**Proof.** We first determine the functions \( \psi_{j,k} \) such that

\[
\sum_{j=1}^{n} \sum_{k=1}^{m} z_{j,k} \left( \psi_{j,k} - \frac{\partial \alpha_{j,k-1}}{\partial \theta_j} \right) \leq 0. \tag{25}
\]

In light of the definition of \( \hat{\theta}_j \) and 0 < \( S \cdot (\cdot) \) \((\cdot) \leq L(L is the number of neural network weights), a straightforward calculation shows that

\[
\sum_{j=1}^{n} \sum_{k=1}^{m} z_{j,k} \left( \psi_{j,k} - \frac{\partial \alpha_{j,k-1}}{\partial \theta_j} \right) \leq \sum_{j=1}^{n} \sum_{k=1}^{m} \left( b_j \hat{\theta}_j - \frac{t \alpha_{j,k-1}}{\theta_j} \right)
\]

Thus, by choosing \( \psi_{j,k} \) as

\[
\psi_{j,k} = -b_j \hat{\theta}_j - \frac{t \alpha_{j,k-1}}{\theta_j}
\]

(25) holds. Similarly, the following can be verified easily.

\[
\sum_{j=1}^{n} g_0 \psi_{j,k} \left( \frac{r_j}{2} \right) \left( \frac{t \alpha_{j,k-1}}{\theta_j} \right) \leq \sum_{j=1}^{n} g_0 \psi_{j,k} \left( \frac{r_j}{2} \right) \left( -\hat{\theta}_j \right)^2 \tag{26}
\]

At the present stage, choose Lyapunov functional as \( V = V_{n,m} \). Then, combining (24)–(26) results in

\[
\dot{V} \leq -\sum_{j=1}^{n} \sum_{k=1}^{m} j_{j,k} g_0 \psi_{j,k} - \sum_{j=1}^{n} g_0 \psi_{j,k} \left( \frac{r_j}{2} \right) \left( -\hat{\theta}_j \right)^2 + C
\]

\[
+ \sum_{j=1}^{n} \sum_{k=1}^{m} \left( 1 - 2 \tanh^2 \left( \frac{z_{j,k}}{\eta_{j,k}} \right) \right) U_{j,k}(x)
\]

where \( C = \sum_{j=1}^{n} \sum_{k=1}^{m} \frac{1}{2} \left( \alpha_{j,k}^2 + \alpha_{j,k}^2 \right) \) and \( \sum_{j=1}^{n} \sum_{k=1}^{m} \frac{g_0}{\eta_{j,k}} \hat{\theta}_j^2 \) is a constant. Thus, by (27) the boundedness follows immediately following the same line used in the proof of Theorem 1 in Ge and Tee (2007) and Zhou et al. (2005). The proof is thus completed. □

5. Simulation example

In this section, a simulation example is used to illustrate the effectiveness of the proposed adaptive neural control method. Consider the following nonlinear time-delay system.

\[
\dot{x}_{1,t} = -x_{1} + (1 + \sin^2(x_1) + x_1^2)
\]

\[
\dot{x}_{1,t+1} = x_1 x_{1,t} + x_{1,t} + x_{2,t} + (1 + \sin^2(x_1) + 0.5 \cos^2(x_2,2)) u_{1,t} + x_{r_1,t}.
\]

\[
\dot{x}_{2,t} = (x_1 + x_{2,t}) x_{2,t} - x_1 u_{1,t} + (2 + \sin^2(u_1)) - \sin(x_1 x_2 - x_1,1) u_{2,t} + x_{r_2,t},
\]

where \( x_{j,i} = x_{j,i}(t - \tau_{j,i}) \), for \( j = 1, 2, i = 1, 2 \), and the time-delays are chosen as \( \tau_{1,1} = 2, \tau_{1,2} = 1.5, \tau_{2,1} = 0.5, \) and \( \tau_{2,2} = 1 \).

Clearly, if remove the terms \( 0.5 \cos^2(x_2,2) u_{1,t} \) and \( \sin^2(u_1) \) in \( g_{1,2} \) and \( g_{2,2} \), respectively, this system is the same as the one used in Ge and Tee (2007). Now, because \( g_{1,2} \) contains \( x_{2,t} \) and \( g_{2,2} \) contains \( u_t \), the neural control scheme proposed in Ge and Tee (2007) cannot be used to control this system. Given the reference output signals as \( y_{r_1} = 0.5 \sin(t) + \sin(0.5t) \) and \( y_{r_2} = 0.5 \sin(t) + \sin(0.5t) \), for the control law (22) and the NN adaptation law (5) we choose the design parameters as: \( k_{1,1} = k_{1,2} = k_{2,1} = k_{2,2} = 30, a_{1,1} = a_{1,2} = 3, a_{2,1} = a_{2,2} = 1, r_1 = r_2 = 600, b_1 = b_2 = 0.075 \).

The simulation is run under the initial conditions \( s_j(\theta) = 0 \) for \( -\tau_{j,i} \leq \theta \leq 0, j = 1, 2, i = 1, 2, \) and \( [\hat{\theta}_1(0), \hat{\theta}_2(0)] = [0, 0]^T \). The simulation results are shown in Figs. 1–4. Figs. 1 and 2 show the corresponding system outputs and the reference signals. Fig. 3 shows the responses of state variables \( x_{1,t} \) and \( x_{2,t} \). Fig. 3 displays the control input signals \( u_1 \) and \( u_2 \). Fig. 4 shows the boundedness of adaptive parameters \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \). From the simulation results, it can clearly be seen that the proposed controller guarantees the boundedness of all the signals in the closed-loop system, and also achieves the good tracking performance. In addition, to control this system, our method requires only two adaptation laws.

6. Conclusion

A novel adaptive neural network tracking control design scheme has been proposed for a class of MIMO nonlinear time-delay systems with block-triangular structure. The suggested control law guarantees that the tracking errors converge to a neighborhood of the origin and all the other signals in the resulting closed-loop system remain bounded. Compared with the existing
results, the main advantage of our result is that the restriction on the control gain functions has been removed and the number of adaptation parameters has been considerably reduced.

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