Comparison of Two Auto-Tuning Methods for a Variable Stiffness Vibration Absorber

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Abstract

A tunable vibration absorber is developed and its stiffness can be varied on-line. The absorber system is mounted on a clamped-clamped beam acting as a primary system. The objective is to suppress vibration of the primary beam subject to a harmonic excitation whose frequency may vary. A system modeling is conducted. The frequency response of the system is given to show the operating range of the absorber system. Using a simplified two-degree-of-freedom model, two auto-tuning methods are studied. The methods differ in the way of how to identify the exciting frequency. The first method follows a common practice that uses the frequency of the maximum peak in the response spectrum as the exciting frequency. The second method makes use of information of both the response spectrum and the natural frequencies. An experimental study is conducted to compare the two methods. The study has shown that the second method performs better than the first method in terms of frequency tracking ability and robustness to disturbance.

La comparaison de Deux Méthodes d'Auto-Accordant pour un Absorbeur de Vibration de Raideur Variable

Résumé

Un absorbeur variable de vibration de raideur est développé pour étudier en ligne accordant. Le système d'absorbeur est monté sur un rayon serré-serré sert d'un système primaire. L'objectif sera obligé à éliminer la vibration du sujet de rayon primaire à une excitation harmonique dont la fréquence peut varier. Un modelage de système est dirigé. La réponse de fréquence du système est donnée pour montrer la gamme d'opération du système d'absorbeur. L'utilisation d'un a simplifié de deux degrés-de-le modèle de liberté, deux méthodes d'auto-accordement sont étudiées. Les méthodes diffèrent dans la façon de comment identifier la fréquence passionnante. La première méthode suit une pratique commune qui utilise la fréquence du sommet maximum dans la gamme de réponse comme la fréquence passionnante. La deuxième méthode utilise l'information de la gamme de réponse et les fréquences naturelles. Une étude expérimentale est dirigée pour comparer les deux méthodes. L'étude a montré que la deuxième méthode exécute mieux que la première méthode sur le plan de la fréquence traquant la capacité et la robustesse au dérangement.
1. Introduction

A vibration absorber is a passive device used to suppress vibration of a machine excited by a harmonic force. In theory, when the absorber's natural frequency is equal to the exciting frequency, vibration of the machine or primary system is completely eliminated. However, if the exciting frequency varies, vibration of the entire system may increase significantly. One of the solutions to the problem is to make the absorber's frequency tunable on-line. Tunable vibration absorbers belong to semi-active or adaptive-passive vibration control. As reactive forces are used, semi-active control devices consume less power than active control devices. Because of adaptability, the performance of tunable vibration absorbers will not degrade due to a varying environment. Moreover, they can behave as passive devices in the event of loss of power; thus are reliable.

Several tunable vibration absorbers have been proposed. In [1], a variable stiffness vibration absorber was developed. Variation of the absorber stiffness was achieved by varying the effective number of coils in a helical spring used as the absorber stiffness. The same vibration absorber was used to study non-collocated adaptive-passive vibration control in [2]. A temperature dependent viscoelastic material was used to design a vibration absorber with an adaptable suppression band in [3]. A vibration absorber developed in [4] consists of a flexible cantilever beam attached by a mass at its free end. By varying the length of the beam, the absorber frequency can be varied. A variable stiffness device proposed in [5] has four coil springs arranged in a plane rhombus configuration. The aspect ratio of the rhombus configuration can be varied by a linear electromechanical actuator to achieve a continuous variation of the absorber stiffness. In [6], shape memory alloy (SMA) spring elements were used to construct an adaptive tuned vibration absorber. The elastic modulus of the SMA elements may change by a factor as high as three by resistive heating. However, it was pointed out that continuous control of the elastic modulus of SMA is difficult when temperature is the only control input. A variable stiffness absorber similar to the one developed in [4] was used to compare with a variable damping absorber in [7]. A commonly used tuning method [1, 2, 4] was used in the study. The main idea of the method is to use the frequency corresponding to the maximum peak in the spectrum of the measured response as the exciting frequency and then to vary the absorber stiffness such that the absorber frequency coincides the measured exciting frequency. The study revealed that such a strategy does not always result in a correct adjustment of the absorber frequency. A second method was proposed in [8] to overcome the aforementioned problem. The method was tested using computer simulation. As a follow-up, this paper further justifies the rationale of the second tuning method and reports the results of an experimental comparison study.

![Figure 1. Variable stiffness vibration absorber](image-url)
Figure 2. Experimental set-up

Figure 1 shows a photo of the vibration absorber used in the study. Figure 2 shows the entire experimental setup that consists of three subsystems: absorber, primary beam, and computer control system. The main part of the absorber system is a cantilever beam (8) and its free end is attached by an absorber mass (9). The beam length can be varied by moving a movable support (7). The movable support is driven by two lead screws (5). The lower lead screw is driven by a DC motor (3) and through a pulley-belt set (4), the upper lead screw rotates simultaneously. An encoder (6) is attached to the upper lead screw to measure the position of the movable support. The absorber beam is an aluminum rod of 6.35 mm in diameter. The range of the beam length variation is 306 mm. The motor is a 12V DC permanent magnet reversible motor and its no-load speed is 180.1 RPM and full load speed is 135.6 RPM at a torque of 19.28 oz-in. The encoder is HEDS-7500 series with a resolution of 256 pulses per revolution. An electromagnetic shaker (1) is used to generate a non-contact exciting force. A small permanent magnetic plate (2) is glued on the beam to interact with the electromagnetic force. The shaker is driven by a power amplifier (B&K 2706). A Pentium III (550MHz) PC computer is used to control the system. The Data Acquisition (DAQ) Board is DS1102 from dSPACE. ControlDesk (dSpace) is used to interface between Matlab, Simulink and DS1102. The primary beam (10) is a clamped-clamped beam made of aluminum. The dimension of the beam is 50.8 mm (width) × 5.08 mm (thickness) × 1057 mm (length). Vibration of the primary beam is measured by an accelerometer (B&K 4393V) (11). This signal is used for on-line tuning. Another accelerometer (12) is placed on the absorber mass and its signal is used for comparison. A charge amplifier (B&K Nexus2692) is used to condition the accelerometer signals. A motor control command is generated by the computer and sent to a motor driver circuitry to regulate the current from a DC power supply.

2. MODELING

Figure 3(a) shows a schematic of the model used in the analysis. The transverse displacement $w(\xi,t)$ of the primary beam is a function of both the axial position $\xi$ and the time $t$. Applying Newton’s second law of motion to a differential beam element results in
\[
\mu \ddot{w}(\xi, t) + \gamma \dot{w}(\xi, t) + EIw'''(\xi, t) = F_0 \sin \omega t \delta(\xi - l_f) + \\
\{k_a [u(t) - w(l_a, t)] + c_a \ddot{u}(t) - \dot{w}(l_a, t) - m_0 \ddot{w}(l_a, t)\} \delta(\xi - l_a)
\]

(1)

where \( \mu \) is the mass per unit beam length, \( \gamma \) the damping value per unit beam length, \( EI \) the flexural rigidity of the beam, \( F_0 \) the amplitude of the exciting force, \( \omega \) the exciting frequency, \( k_a \) the stiffness of the absorber, \( c_a \) the damping value of the absorber, \( m_0 \) the mass of the absorber frame, \( u \) the position of the absorber mass, \( \delta(\xi - l_f) \) and \( \delta(\xi - l_a) \) are Dirac delta functions. In Eq. (1), primes and dots denote partial derivatives with respect to \( \xi \) and \( t \), respectively. Applying Newton’s second law to the absorber mass results in

\[
m_a \ddot{u}(t) + c_a [\ddot{u}(t) - \dot{w}(l_a, t)] + k_a [u(t) - w(l_a, t)] = 0
\]

(2)

where \( m_a \) is the absorber mass.

Figure 3. Schematics of the model: (a) primary beam and absorber; (b) 2-DOF system

To transfer the above equations to a set of ordinary differential equations, the mode summation method is used. The first three modes of the primary beam’s vibration are considered, i.e.,

\[
w(\xi, t) = \Phi(\xi) q(t)
\]

(3)

where

\[
\Phi(\xi) = [\phi_1(\xi) \quad \phi_2(\xi) \quad \phi_3(\xi)], \quad q(t) = [q_1(t) \quad q_2(t) \quad q_3(t)]^T
\]

with \( \phi_i(\xi) \) representing the \( i \)th mode shape of a clamped-clamped beam and \( q_i(t) \) representing the \( i \)th generalized coordinate [9]. After some manipulation [7], the following equation is obtained

\[
M \dddot{q} + \dot{D} \ddot{q} + K q = Bf
\]

(4)
where \( M \) is the mass matrix, \( D \) the damping matrix, \( K \) the stiffness matrix, \( B_i \) the input matrix, \( f \) the exciting force and they are of the forms

\[
M = \begin{bmatrix} m_0 \Phi^T(l_a) \Phi(l_a) + \mu l I_{3 \times 3} & 0_{3 \times 1} \\ 0_{3 \times 3} & m_a \end{bmatrix}, \quad D = \begin{bmatrix} \gamma l I_{3 \times 3} + c_a \Phi^T(l_a) \Phi(l_a) & -c_a \Phi^T(l_a) \\ -c_a \Phi(l_a) & c_a \end{bmatrix}, \quad K = \begin{bmatrix} E I l \Lambda + k_a \Phi^T(l_a) \Phi(l_a) & -k_a \Phi^T(l_a) \\ -k_a \Phi(l_a) & k_a \end{bmatrix}, \quad B_i = \begin{bmatrix} \Phi^T(l_i) \\ 0 \end{bmatrix}, \quad f = F_0 \sin \omega t.
\]

In the above equations, \( I_{i \times i} \) denotes an \( i \times i \) identity matrix, \( 0_{i \times j} \) an \( i \times j \) zero matrix, \( \Lambda = \text{diag}[\beta_i^T] \) with constants \( \beta_i, i = 1, 2, 3 \) [9]. Equation (4) can be transferred into a state-space representation by defining a state vector \( x = [q^T \quad u \quad q^T \quad u]^T \):

\[
\dot{x}(t) = Ax(t) + Bf(t) \\
y(t) = w(l_s, t) = Cx(t)
\]

where

\[
A = \begin{bmatrix} 0_{4 \times 4} & I_{4 \times 4} \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0_{4 \times 1} \\ M^{-1}B_i \end{bmatrix}, \quad C = [\phi_1(l_s) \quad \phi_2(l_s) \quad \phi_3(l_s) \quad 0_{5 \times 1}]
\]

with \( l_s \) denoting the location of observation. The transfer function is given as

\[
H(s) = Y(s)/f(s) = C(sI - A)^{-1}B.
\]

Figure 4. Frequency responses of the beam at \( \xi = l_a \) for the smallest \( f_a \) and the largest \( f_a \)

Let \( s = j \omega \) in Eq. (6), the frequency response \( H(j \omega) \) can be obtained. The following data are used: \( \mu l = .7638 \) kg, \( EI = 39.52 \) Nm², \( m_0 = .8887 \) kg, \( m_a = .1770 \) kg, \( l_a = l_s = 528.5 \) mm, \( l_f = 247 \) mm.
$\gamma = 0.2142 \text{ kg/s/m, } c_a = 0 \text{ kg/s, and } f_a = 11.79 \text{ Hz with } f_a = \sqrt{k_a/m_a/(2\pi)}$ denoting the absorber frequency. The natural frequencies of the entire system are found to be 9.73 Hz, 14.29 Hz, 64.9 Hz, and 101.79 Hz. Figure 4 shows the frequency response magnitudes corresponding to the smallest absorber frequency $f_a = 6.6 \text{ Hz}$ and the largest absorber frequency $f_a = 13.5 \text{ Hz}$, respectively.

3. AUTO-TUNING METHOD ONE

As shown in the previous section, the objective of auto-tuning is to make the absorber frequency equal the exciting frequency, i.e., $f_a = f$. The previous analysis has shown that the system behaves strongly like a 2-DOF system in the adjustable range of the absorber frequency. Therefore a 2-DOF system shown in Fig. 3(b) is used in the following study. Equations of motion for the simplified system are given as

$$m\ddot{x} + (c + c_a)\dot{x} - c_a\dot{x}_a + (k + k_a)x - k_a x_a = F_0 \sin \omega t$$
$$m_a\ddot{x}_a - c_a\dot{x} + c_a\dot{x}_a - k_a x + k_a x_a = 0$$

where $m = 1.653 \text{ kg is the sum of the mass } \mu l \text{ of the primary beam and the absorber frame } m_0$, $k = 192EI/l^2 = 6426 \text{ N/m is the equivalent stiffness of the primary beam, the damping values } c = 1.025 \text{ kg/s, } c_a = 1.1110 \text{ kg/s. Note that as the purpose of this simplified model is to have a similar 2-DOF system for testing of the tuning algorithms, the values used may not be the closest approximation of the real system. For example, the effective lumped mass of the beam should be } 0.4857\mu l 	ext{ if an energy formulation is used. The damping values are chosen such that the damping ratios are } \zeta = c/(2\sqrt{k/m}) = 0.005 \text{ and } \zeta_a = c_a/(2m_0 \omega_p) = 0.005 \text{ with } \omega_p = \sqrt{k/m}.$

To measure the magnitude of response, a root mean squared value of the responses is defined as

$$\text{rms}_x = \sqrt{\sum_{i=1}^{I} x^2(i)/I}$$

where $x(i)$ is the sampled response and $I$ is the total number of samples in the time interval of interest. A vibration attenuation measure is defined as

$$\delta = -20 \log_{10} \frac{\text{rms}_x^c}{\text{rms}_x^n} \text{ dB}$$

where $\text{rms}_x^c$ and $\text{rms}_x^n$ are the root mean squared values for the case with auto-tuning control and the case without auto-tuning control, respectively.

The first tuning method is based on one frequency from the spectrum of the response of the primary mass. It contains the following steps:

1. Sample the response $x$ of the primary mass over a specified time period at a sampling interval $\Delta t$.
   The time period of each group of the data is chosen to be $T = N\Delta t$ where $N$ is the number of data.
2. Compute \( \text{rms}_x \) of the sampled response. Compare \( \text{rms}_x \) with a prescribed vibration magnitude threshold value \( \text{rms}'_x \) if \( \text{rms}_x < \text{rms}'_x \), go back Step 1. Otherwise, continue to Step 3.

3. Obtain the spectrum of the response by applying the Fast Fourier Transform (FFT) to the sampled responses and find the frequency corresponding to the maximum magnitude of the spectrum. Let this frequency be the new measured exciting frequency \( f^\text{new} \).

4. Compare \( f^\text{new} \) with the absorber frequency \( f_a \) in the last iteration, if \( |f^\text{new} - f_a| > f_{tol} \), where \( f_{tol} \) is a prescribed frequency tolerance, set the absorber frequency equal the measured exciting frequency or \( f_a = f^\text{new} \), go to Step 5. Otherwise keep \( f_a \) unchanged, go to Step 1.

5. Calculate a new absorber stiffness \( k_a^\text{new} = (2\pi f^\text{new})^2 m_x \), update \( k_a \) with \( k_a^\text{new} \) and go to Step 1.

A Simulink model was built to test the algorithm’s ability to track change in the exciting frequency. To use the Simulink program, one needs to choose the number of data for FFT. The main considerations for it are the frequency resolution and the FFT computation time. The sampling time used is \( \Delta t = .001 \) s. Thus, the frequency resolution is \( \Delta f = 1000/N \) Hz. If \( N = 1024 \) is used, \( \Delta f = .977 \) Hz. If \( N = 2048 \), \( \Delta f = .488 \) Hz. A data length of 2048 is considered to be a proper compromise between a sufficient frequency resolution and a reasonable computation time. A testing scenario is devised. The exciting frequency is 10 Hz during the time interval \( 0 \leq t < 10 \) s, at the moment of 10 s, the exciting frequency is changed to be 8.5 Hz. In the interval \( 0 \leq t < 10 \) s, the absorber is tuned such that \( f_a = 10 \) Hz. The natural frequencies of the entire system are 8.478 Hz and 11.77 Hz, respectively. Therefore, the sudden change in the exciting frequency causes the system near resonance after \( t > 10 \) s. Figure 5 shows the response of the primary mass and frequency tracking. The response magnitude threshold is \( \text{rms}'_x = 2 \times 10^{-4} \) that is about the vibration level achieved when the absorber is tuned. Shown in the figure, at \( t = 10 \) s, the response is increased due to resonance. After the absorber frequency is adjusted to be the measured exciting frequency at \( t = 12.048 \) s, the response starts to diminish. The figure also shows some interesting behaviors. In the beginning, the measured frequency is not close to the true exciting frequency. However, the absorber frequency is not updated because \( \text{rms}_x < \text{rms}'_x \). After \( t \geq 12.048 \) s, the measured frequency drifts away from the true exciting frequency and the absorber frequency is updated at each step until \( \text{rms}_x < \text{rms}'_x \) when \( t > 26.624 \) s.

![Figure 5. Tuning results with \( f_{tol} = .4 \) (Hz): (a) response of the primary mass; (b) frequencies](image-url)
4. AUTO-TUNING METHOD TWO

The above simulation indicates that the use of the frequency corresponding to the maximum FFT magnitude as the exciting frequency may result in an incorrect adjustment of the absorber frequency. Whenever the exciting frequency changes, the general responses of the primary mass and the absorber mass are given as

\[ x(t) = x_p(t) + x_h(t), \quad x_a(t) = x_{ap}(t) + x_{ah}(t) \] (10)

where \( x_p \) and \( x_{ap} \) are the steady state responses and \( x_h \) and \( x_{ah} \) are the transient responses. The steady state responses are of the form

\[ x_p(t) = A\sin(\omega t - \theta), \quad x_{ap}(t) = A_p\sin(\omega t - \theta_p) \] (11)

where \( A \) and \( \theta \) denote the amplitude and phase of the steady state response of the primary mass, respectively, \( A_p \) and \( \theta_p \) denote the amplitude and phase of the steady state response of the absorber mass, respectively. The following analysis intends to compare the peaks in the FFT spectrum and the amplitudes of the steady and transient responses. To simplify the analysis, it is assumed that the system damping is proportional [9]. Therefore, the transient responses are of the form

\[ x_h(t) = b_1e^{-\zeta_1\omega_1 t}\sin(\omega_1t+\phi_1) + b_2e^{-\zeta_2\omega_2 t}\sin(\omega_2t+\phi_2) \]

\[ x_{ah}(t) = b_1u_1e^{-\zeta_1\omega_1 t}\sin(\omega_1t+\phi_1) + b_2u_2e^{-\zeta_2\omega_2 t}\sin(\omega_2t+\phi_2) \] (12)

where \( \zeta_i, \omega_n, \) and \( \omega_n = \omega_n\sqrt{1-\zeta_i^2}, i=1,2 \) are the damping ratio, natural frequency, and damped natural frequency, \( b_i, \phi_i, i=1,2 \) are the amplitude and phase of the transient response, \( u_i, i=1,2 \) is the ratio of the mode shape.

With the model used in the previous section, it is possible to determine the constants in Eqs. (11) and (12) if a testing scenario is chosen. For this purpose, it is assumed that the absorber is tuned such that \( f = f_a = f_p = 10.05 \) Hz where \( f_p = \sqrt{k/m}/(2\pi) \) and the system responses are in a steady state when \( t < t_1 = 10 \) s. Note that the natural frequencies of the entire system are \( f_{n1} = 8.52 \) Hz, \( f_{n2} = 11.90 \) Hz. At \( t = 10 \) s, the exciting frequency is changed to be a new value. After \( T = 2.048 \) s or at \( t_2 = t_1 + T = 12.048 \) s, the absorber frequency is tuned again according to the peak frequency in the FFT spectrum. To distinguish these three periods, \( 0 \leq t < t_1 \) is referred to as the initial period denoted by superscript \( 'i' \), \( t_1 \leq t < t_2 \) the middle period denoted by superscript \( 'm' \), and \( t_2 < t \) the final period denoted by superscript \( 'f' \). The constants \( b_i, \phi_i, i=1,2 \) for the middle period can be determined by solving the following equations

\[ x_h''(t_1) = x_p''(t_1) - x_{ap}''(t_1), x_{ah}''(t_1) = x_{ap}''(t_1) - x_{ah}''(t_1) \]

\[ x_h''(t_1) = \dot{x}_p''(t_1) - \dot{x}_{ap}''(t_1), \dot{x}_{ah}''(t_1) = \dot{x}_{ap}''(t_1) - \dot{x}_{ah}''(t_1) \] (13)

The constants \( b_i, \phi_i, i=1,2 \) for the final period can be determined by solving the following equations
\[ x_i^f(t) = x_i^a(t) - x_i^f(t), x_i^a(t) = x_i^a(t) - x_i^b(t) \]
\[ \dot{x}_i^f(t) = \dot{x}_i^a(t) - \dot{x}_i^f(t), \dot{x}_i^a(t) = \dot{x}_i^a(t) - \dot{x}_i^b(t) \]

(14)

Two testing scenarios are considered. For scenario one, the exciting frequency is changed to be \( f = 8.52 \) Hz at \( t_1 \) and the absorber frequency is adjusted to be \( f_a = 8.79 \) Hz at \( t_2 \). For scenario two, the exciting frequency is changed to be \( f = 5.11 \) Hz at \( t_1 \) and the absorber frequency is adjusted to be \( f = 11.7 \) Hz at \( t_2 \). The damping values \( c \) and \( c_a \) are chosen such that the system has a proportional damping with a modal damping ratio of 0.005. Table 1 lists the results. In the table, \( \bar{b}_1 \) and \( \bar{b}_2 \) for the middle period are evaluated by

\[ \bar{b}_i = (b_i e^{-\zeta_1 \omega_n (t_1 + T)} + b_i e^{-\zeta_2 \omega_n (t_1 + T)}) / 2 \]

(15)

and \( \bar{b}_1 \) and \( \bar{b}_2 \) for the final period are evaluated with \( t_1 \) replaced by \( t_2 \). \( X_i, i = 1, 2 \) is the magnitude of the FFT spectrum of the total response of the primary mass and \( f_i, i = 1, 2 \) is the corresponding peak frequency. Indicated in Table one, for scenario one, during the middle period, the first peak frequency is associated with the exciting frequency while the second one is associated with the natural frequency. Tuning based on the peak frequency at the end of the middle period is correct. During the final period, both the peak frequencies are associated with the natural frequencies. Because the absorber is tuned, the amplitude of the steady state response is the minimum. However, the amplitudes of the transient responses are relatively large at least for a short period if the system damping is low. If the first peak frequency is still taken as the exciting frequency, an incorrect tuning occurs. For scenario two, during the middle period, three peak frequencies are associated with the second natural frequency, the exciting frequency, and the first natural frequency, respectively. If the absorber frequency is set to be the first peak frequency, mistuning results and the steady state response remains almost unchanged.

Table 1. Constants for the general responses and FFT magnitudes and frequencies

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Period</th>
<th>( A )</th>
<th>( \bar{b}_1 )</th>
<th>( \bar{b}_2 )</th>
<th>( X_1 / f_1 )</th>
<th>( X_2 / f_2 )</th>
<th>( X_2 / f_2 )</th>
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</thead>
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<tr>
<td>initial</td>
<td></td>
<td>7.74E-6</td>
<td>0</td>
<td>0</td>
<td>5.91E-6/10.3</td>
<td>N/A</td>
<td>N/A</td>
</tr>
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<td>Middle</td>
<td>3.24E-3</td>
<td>2.76E-3</td>
<td>1.48E-4</td>
<td>4.83E-1/8.79</td>
<td>1.04E-1/11.7</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>Final</td>
<td>1.49E-4</td>
<td>6.44E-3</td>
<td>2.57E-4</td>
<td>5.31E-1/7.81</td>
<td>2.18E-1/11.2</td>
<td>N/A</td>
</tr>
<tr>
<td>Two</td>
<td>Middle</td>
<td>2.21E-4</td>
<td>1.90E-4</td>
<td>1.69E-4</td>
<td>1.55E-1/11.7</td>
<td>1.53E-1/4.88</td>
<td>1.30E-1/8.30</td>
</tr>
<tr>
<td></td>
<td>Final</td>
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<td>1.32E-4</td>
<td>6.47E-5</td>
<td>1.54E-1/4.88</td>
<td>1.15E-1/8.79</td>
<td>5.84E-2/13.2</td>
</tr>
</tbody>
</table>

If the natural frequencies of the system are available, they can be used to compare with the peak frequencies to select a frequency that is more likely to be associated with the exciting frequency. This prompts the second tuning method. The procedure of the second tuning method is the same as the first tuning method except that the measured exciting frequency is determined as follows. By neglecting damping in the system, the natural frequencies can be found to be \( f_{n1} \) and \( f_{n2} \) by using Eq. (5.26) in [9] where \( f_{n1} < f_{n2} \). Three possibilities are considered in terms of the peak frequencies over the frequency range of interest in the spectrum.

1. If there is only one peak frequency, this frequency is used as the measured exciting frequency.
2. If there are two peak frequencies \( f_1 \) and \( f_2 \) with \( X_1 > X_2 \), compare them with \( f_{n1} \) and \( f_{n2} \)

\[ \Delta f_i = \min(|f_i - f_{n1}|, |f_i - f_{n2}|), i = 1, 2 \]

(16)
If $\Delta f_1 < \Delta f_{\text{sol}}$ and $\Delta f_1 < \Delta f_{\text{sol}}$ where $\Delta f_{\text{sol}}$ is a prescribed number, the measured exciting frequency is determined by

$$f_{\text{new}} = \frac{(X_1 f_1 + X_2 f_2)}{(X_1 + X_2)}.$$ (17)

Otherwise, the measured exciting frequency is chosen to be the one that is farther away from $f_{n1}$ and $f_{n2}$.

3. If there are three peak frequencies $f_1$, $f_2$, and $f_3$ with $X_1 > X_2 > X_3$, compare them with $f_{n1}$ and $f_{n2}$, i.e.,

$$\Delta f_i = \min(|f_i - f_{n1}|, |f_i - f_{n2}|), \quad i = 1, 2, 3$$
$$\Delta f_m = \max(\Delta f_1, \Delta f_2, \Delta f_3).$$ (18)

If $\Delta f_m > \Delta f_{\text{sol}}$, $f_{\text{new}} = f_m$ where $f_m$ is the frequency corresponding to $\Delta f_m$. Otherwise, $f_{\text{new}}$ is determined by Eq. (17).

The rationale for Step 2 is that if the peak frequencies are close enough to the natural frequencies, a weighted frequency is used as the measured exciting frequency. The rationale for Step 3 is that the peak frequency farthest away from the natural frequencies is more likely to be the exciting frequency.

Figure 6 shows the results when the measured exciting frequency is determined using the second method. It can be seen that the measured exciting frequency is very close to the true one during the interval of $2.048 < t < 10.24$ s. After $t > 22.528$ s, the measured exciting frequency follows the true one closely.

![Figure 6](image.png)

Figure 6. Tuning results using the second method with $f_{\text{sol}} = .4$ Hz and $\Delta f_{\text{sol}} = .4$ Hz: (a) response of the primary mass; (b) frequencies

To further compare the results with different tuning parameters, a frequency tracking measure is defined as

$$\text{rms}_f = \sqrt{\frac{\sum_{i=1}^l [f(i) - f_s(i)]^2}{l}}$$ (19)
where $f(i)$ and $f_s(i)$ denote the exciting frequency and the absorber frequency in the $i$th FFT sampling interval, respectively. Table 2 lists the results. In the table, two values are evaluated for the attenuation measure. The first value is based on the response for the entire duration or $0 \leq t \leq 40$ s and the second value is based on the response for the ending period or $25 \leq t \leq 40$ s. In the same way, two values are evaluated for the frequency tracking error measure. Thus the second values measure the tuning quality long after the change of the exciting frequency. When the second method is used, $f_{tol} = .4$ Hz. It can be seen that with $N = 2048$, the two methods result in a similar vibration attenuation measured by the first value. However, when measured by the second value, the second method performs better than the first one. Moreover, the second method achieves a smaller tracking error in terms of both the first and second values. With $N = 1024$, the second method gives a better result when $f_{tol} = .15$ Hz is used. Figure 7 compares the frequency tracking for the case of $N = 1024$ and $f_{tol} = .15$ Hz. It is noted that the first method causes an unstable tuning.
Table 2. Comparison of the results of the two tuning methods
5. EXPERIMENT

This section reports the comparison of the two tuning methods experimentally. The objective of the on-line tuning is to move the movable support to the position such that the absorber frequency equals the measured exciting frequency. Because the absorber beam is not an ideal cantilever beam, the relationship between the absorber beam length and the absorber frequency was determined experimentally. With the absorber system firmly fixed to the ground, an impact was applied to the absorber mass and the response was recorded using an accelerometer placed on the absorber mass. The peak frequency was found from the FFT spectrum of the response. The peak frequency was used as the absorber frequency corresponding to the beam length. Experiments were repeated by increasing the beam length by a small amount. A fourth order polynomial was used to curve-fit the testing results. The obtained equation is of the form

\[ l_b = 320 + 210f_a - 60.5f_a^2 + 5.38f_a^3 - 0.16f_a^4 \]  

(20)

where \( l_b \) is the absorber beam length. The natural frequencies of the entire system corresponding to different absorber beam lengths were also determined in a similar way. The obtained equations are given by

\[ f_{n1} = 7.59 + 8.72 \times 10^{-4}l_b - 6.43 \times 10^{-6}l_b^2 - 8.06 \times 10^{-8}l_b^3 + 1.72 \times 10^{-10}l_b^4 \]

\[ f_{n2} = 11.9 + 9.38 \times 10^{-2}l_b - 1.19 \times 10^{-3}l_b^2 + 4.41 \times 10^{-6}l_b^3 - 5.40 \times 10^{-9}l_b^4. \]  

(21)
Figure 8 shows a Simulink model that implements auto-tuning. The acceleration signal of the primary beam is acquired through channel ADC#1. The base sampling time is 1 ms. An S function named as “buffer1” is used to collect the data for a specified length such as $N = 1024$ or 2048. The sampled data are held by a Zero-Order Hold (ZOH) with a sampling time $t_{ZOH} = 0.001N$ s. Then the sampled data are transferred to an S function named “absorber frequency”. The reading of the encoder is input through ENC_POS #2. The reading is converted into the position of the movable plate by an S function “plate position”. An S function “buffer2” collects a group of the values from “plate position”. The values are held by a ZOH before transferred to an S function “natural frequencies”. In “natural frequencies”, the average absorber beam length is calculated and used to find the natural frequencies based on Eq. (21). The S function “absorber frequency” takes in a group of the sampled accelerometer signals, the computed natural frequencies, and the absorber frequency of the previous iteration. The function conducts the FFT computation and identifies dominant peak frequencies. With the peak frequencies and the natural frequencies available, the absorber frequency is determined according to the tuning algorithms. The absorber frequency is sent to a function where the desired absorber beam length is computed using Eq. (20). The value is passed to a Unit Delay with a sampling time of $0.001N$ s in order to transfer the value from the slow block back to the fast block. The desired position of the movable plate is found by subtracting the desired absorber beam length from the maximum absorber beam length 360 mm. The desired position of the movable plate is compared with the actual position to find the position error. The position error is sent to an S function “motor control” which implements a simple proportional control with a saturation. Then the command is output to the motor driver via DAC #2. A sinusoidal signal is sent to the power amplifier for the electromagnetic shaker via DAC #1.

Three testing scenarios were considered, namely, single-step change, impact disturbance, and multi-step change. For the single-step change, the system was excited at a frequency of 7 Hz and the absorber frequency was tuned to be this frequency. At 10 second, the exciting frequency was changed to be 8.6 Hz which was close to the second natural frequency. Figure 9 shows the results when the first tuning method was used and the user prescribed parameters were $N = 2048$, $rms_x^t = 7E-2$, and $f_{sol} = .1$ Hz. It can be seen that at $t = 10$ s, the response experiences an increase and starts to decay around 17 second, eventually resumes the minimum level. It is noted that after the absorber is almost tuned, the absorber frequency starts to drift away from the true exciting frequency as the dominant peak no longer corresponds to the exciting frequency. The result by the second tuning method is given in Fig. 10 where
\( \Delta f_{at} = .5 \) Hz was used. It can be seen that the absorber frequency tracks the true exciting frequency closely. The experiments were also conducted with \( N = 1024 \). The results are summarized in Table 3. Overall, the second method results in a greater vibration attenuation and smaller frequency tracking error.

Figure 9. Tuning result using method one for a single-step change: (a) response; (b) frequencies

Figure 10. Tuning result using method two for a single-step change: (a) response; (b) frequencies
To test the algorithm robustness to disturbance, an impact was applied to the beam when the absorber was already tuned after the step change. Figures 11 and 12 show the results using the first method and second method, respectively. It is seen that the first method \( (rms_f = .6690) \) was more sensitive to the disturbance than the second method \( (rms_f = .5107) \).

Figure 11. Tuning result using method one for an impact disturbance: (a) response; (b) frequencies
The last testing scenario was to vary the exciting frequency in a multi-step fashion. A comparison of the frequency tracking results using both the methods is given in Fig. 13. Once again, the second method ($rms_f = .4665$) resulted in a closer and faster frequency tracking than the first method ($rms_f = .7064$).

6. CONCLUSION

A variable stiffness vibration absorber has been developed. The absorber has been used to suppress vibration of a beam structure subject to a sinusoidal excitation. A dynamic modeling has been presented. A simplified 2-DOF system has been used in the auto-tuning study. Two on-line tuning strategies have been developed. The performances of the tuning strategies have been tested using computer simulation. The tuning methods have been evaluated experimentally under three testing scenarios. The following conclusions can be drawn from the study. The absorber is capable of adjusting its stiffness to achieve the best performance in an event of the exciting frequency variation. The use of the FFT peak frequency as the exciting frequency may result in mistuning. The second tuning method...
performs better than the first tuning method in terms of vibration attenuation, frequency tracking, and robustness to disturbance.

**REFERENCE**


