Backscattering enhancement for partially convex targets of large sizes in continuous random media for E-wave incidence

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Abstract

The shape of the target constitutes an important factor in the radar detection problem. In a previous study, the enhancement in the radar cross section (ERCS) has proved to be affected largely by the target parameters as well as the effects of the double passage and the spatial coherence length of incident waves around the target. However, the target size was limited to about less than one wavelength. Here, we estimate numerically the RCS of targets taking large sizes of more than three wavelengths, and analyse the characteristics of RCS. Moreover, we investigate the ERCS phenomenon of such targets under different circumstances of random medium and target configuration. In this regard, we assume partially convex targets in continuous random media and also horizontal incident wave polarization (E-wave incidence).

1. Introduction

The problem of electromagnetic-wave scattering from targets is an interesting field that has been studied by many researchers over many decades. At the beginning, this problem was investigated for targets in free space. A number of methods proposed to formulate the scattering field were presented: examples are in [1–3]. On the other hand, scattering of electromagnetic waves from targets embedded in random media has become of great importance in the fields of radar engineering and remote sensing, in particular under the condition that the backscattering enhancement occurs [4]. The backscattering enhancement produced due to the double passage effect on waves propagating in random media has also attracted considerable attention among condensed-matter physicists, resulting in a large number of publications [5–7].
To study the effects of random media on the RCS of finite-size conducting targets, one of the authors has presented a method for solving the scattering problem as a boundary value problem [8, 9]. Based on this method, numerical results have been shown for RCS of conducting convex bodies such as circular and elliptic cylinders [10]. It has been found that the spatial coherence length (SCL) of the incident wave around the target plays a central role in estimating RCS of convex targets in random media.

Later, we aimed to investigate the effect of target configuration on the enhancement phenomenon in the radar cross section (ERCS). In doing that, we have analysed numerically the ERCS of a perfectly conducting cylinder of nonconvex cross section [11, 12]. We have found that the target configuration, including concavity index and target size, obviously affects the ERCS. Thus, the ERCS of conducting targets in random media is not only dependent on both the effects of double passage and SCL, but it is also extended to take account of target parameters. However, the last conclusion was restricted only to a limited target size of about less than one wavelength as in [11].

To gain a clear understanding of the ERCS of targets with partially convex cross sections in random media, we have decided to evaluate the effects of the target size and curvature on the enhancement factor. To achieve this aim, we draw on our method used previously to conduct numerical results for the RCS of concave–convex targets of large sizes of more than three wavelengths and different SCL. We also investigate the characteristics of RCS of these targets, considering the horizontal polarization (E-wave incidence). Next, we stress the numerical analysis of the ERCS in different regions of target size compared to SCL; the targets’ sizes will be extended to be close to five wavelengths for a deeper investigation and understanding. In the previous work, it has clarified the obvious difference in the behaviour of the ERCS between both the concave and convex illumination portions of concave–convex targets. Each of these regions deserves a separate study; here, we concentrate on the wave backscattering from the convex illumination portion only.

We consider the case where the incident wave becomes incoherent enough around a conducting cylinder although it keeps a finite SCL. The time factor $\exp(-i\omega t)$ is assumed and suppressed in the following section.

2. Formulation

Geometry of the problem is shown in figure 1. A random medium is assumed to be a sphere of radius $L$ around a target of mean size $a \ll L$, and also to be described by the dielectric
constant $\varepsilon(r)$, the magnetic permeability $\mu$ and the electric conductivity $\sigma$. For simplicity $\varepsilon(r)$ is expressed as
\begin{equation}
\varepsilon(r) = \varepsilon_0 [1 + \delta \varepsilon(r)]
\end{equation}
where $\varepsilon_0$ is assumed to be constant and equal to free space permittivity and $\delta \varepsilon(r)$ is a random function with
\begin{equation}
\langle \delta \varepsilon(r) \rangle = 0, \quad \langle \delta \varepsilon(r) \delta \varepsilon(r') \rangle = B(r, r')
\end{equation}
and
\begin{equation}
B(r, r) < 1, \quad k l(r) \gg 1.
\end{equation}
Here, the angular brackets denote the ensemble average, $B(r, r)$, $l(r)$ are the local intensity and local scale-size of the random medium fluctuation, respectively, and $k = \omega \sqrt{\mu_0 \varepsilon_0}$ is the wavenumber in free space. Also $\mu$ and $\sigma$ are assumed to be constant; $\mu = \mu_0$, $\sigma = 0$. For practical turbulent media the condition (3) may be satisfied. Therefore, we can assume the forward scattering approximation and the scalar approximation [13].
Consider the case where a directly incident wave is produced by a line source $f(r')$ distributed uniformly along the $y$ axis. Then, the incident wave is cylindrical and becomes approximately planar around the target because the line source is very far from the target. Here, let us designate the incident wave by $u_{in}(r)$, the scattered wave by $u_s(r)$ and the total wave by $u(r) = u_{in}(r) + u_s(r)$. The target is assumed to be a conducting cylinder whose cross section is expressed by
\begin{equation}
r = a[1 - \delta \cos(\theta - \phi)]
\end{equation}
where $\phi$ is the rotation index and $\delta$ is the concavity index. We can deal with this scattering problem two dimensionally under the condition (3); therefore, we represent $r$ as $r = (x, z)$. Assuming a horizontal polarization of incident waves (E-wave incidence), we can impose the Dirichlet boundary condition for a wave field $u(r)$ on the cylinder surface $S$. That is, $u(r) = 0$, where $u(r)$ represents $E_r$.
According to our method [8–12], using the current generator $Y_E$ and Green’s function in a random medium $G(r|r')$, we can express the surface current wave as
\begin{equation}
J(r_2) = \int_S Y_E(r_2|r_1)u_{in}(r_1|r_1) \, dr_1
\end{equation}
where $r_1$ represents the source point location and it is assumed that $r_1 = (0, z)$ in section 3, and $u_{in}(r_1|r_1) = G(r_1|r_1)$, whose dimension coefficient is understood. Then, the scattered wave is given by
\begin{equation}
u_s(r) = \int_S J_E(r_2)G(r|r_2) \, dr_2
\end{equation}
which can be represented by
\begin{equation}
u_s(r) = \int_S \, dr_1 \int_S G(r|r_2)Y_E(r_2|r_1)u_{in}(r_1|r_1).
\end{equation}
Here, $Y_E$ is the operator that transforms incident waves into surface currents on $S$ and depends only on the scattering body. The current generator can be expressed in terms of wavefunctions that satisfy the Helmholtz equation and the radiation condition. That is, the surface current is obtained as
\begin{equation}
\int_S Y_E(r_2|r_1)u_{in}(r_1|r_1) \, dr_1 \simeq \Phi^{\#}_E(r_2)A^{-1}_E \int_S \langle \Phi^{\#}_E(r_1), u_{in}(r_1|r_1) \rangle \, dr_1
\end{equation}
where
\begin{equation} \int_S \langle \Phi^{\#}_E(r_1), u_{in}(r_1|r_1) \rangle \, dr_1 = \int_S \phi_{in}(r_1) \frac{\partial u_{in}(r_1|r_1)}{\partial n} - \frac{\partial \phi_{in}(r_1)}{\partial n} u_{in}(r_1|r_1) \, dr_1.
\end{equation}
The above equation is sometimes called a ‘reaction’ and named by Rumsey [14]. In (8), the basis functions $\Phi_M$ are called the modal functions and constitute the complete set of wavefunctions satisfying the Helmholtz equation in free space and the radiation condition; $\Phi_M = [\phi_{-N}, \phi_{-N+1}, \ldots, \phi_{m}, \ldots, \phi_N]$, $\Phi_M^*$ and $\Phi_M^T$ denote the complex conjugate and the transposed vectors of $\Phi_M$, respectively, $M = 2N + 1$ is the total mode number, $\phi_m(r) = H_m^{(1)}(kr) \exp(i m \theta)$, and $A_E$ is a positive definite Hermitian matrix given by

$$A_E = \begin{pmatrix} \langle \phi_{-N}, \phi_{-N} \rangle & \cdots & \langle \phi_{-N}, \phi_N \rangle \\ \vdots & \ddots & \vdots \\ \langle \phi_N, \phi_{-N} \rangle & \cdots & \langle \phi_N, \phi_N \rangle \end{pmatrix}$$

In which its $m,n$ element is the inner product of $\phi_m$ and $\phi_n$:

$$\langle \phi_m, \phi_n \rangle = \int_S \phi_m(r) \phi_n^*(r) \, dr.$$ 

The $Y_E$ is proved to converge in the sense of the mean on the true operator when $M \to \infty$.

The average intensity of a backscattering wave for E-wave incidence is given by

$$\langle |u_s(r)|^2 \rangle = \int_S \left( \int \int \int \int \cdots \int \right) \prod_{i \neq j} \langle G(r|r_0)|G(r|r_0)G^*(r|r_{02}) \rangle \langle G(r|r_0)G^*(r|r_{02}) \rangle,$$

$$\times \langle G(r|r_1)|G(r|r_0)G^*(r|r_{02}) \rangle \langle G(r|r_0)G^*(r|r_{02}) \rangle = \prod_{i \neq j} M_{ij}$$

In our representation of $\langle |u_s(r)|^2 \rangle$, we use an approximated solution for the fourth moment of Green’s function in a random medium $M_{22}$. Let us assume that the coherence of waves is kept almost complete at a propagation distance of 2$\pi$, equal to the mean diameter of the cylinder. This assumption is acceptable in practical cases under the condition (3). On the basis of the assumption, it is important here to point out that we are going to present a quantitative discussion for the numerical results in section 3. $M_{22}$ can be obtained as the sum of the second moment products [10, 11, 15] on the assumption that a single point source coincides with a single point observation. That is,

$$M_{22} = \langle G(r|r_1')G(r|r_0)G^*(r|r_{02})G^*(r|r_{02}) \rangle \approx \langle G(r|r_1')G^*(r|r_{02}) \rangle \langle G(r|r_0)G^*(r|r_{02}) \rangle$$

$$+ \langle G(r|r_1')G^*(r|r_{02}) \rangle \langle G(r|r_0)G^*(r|r_{02}) \rangle = M_0(M_0 + M_0)$$

$$M_0 = G_0(r|r_1')G_0(r|r_0)G_0^*(r|r_{02})G_0^*(r|r_{02}) = U \exp(X)$$

$$M_a = \exp(Y_1)$$

$$M_\beta = \exp(Y_2)$$

in which

$$U = \frac{1}{[8\pi k z]^2}$$

$$X = -jk(\gamma_0 - \gamma_{02} + \gamma_{1} - \gamma_{2}) + \frac{jk}{2(z - \gamma_{0})}(x_{01}^2 - x_{02}^2 + x_{1}^2 - x_{2}^2)$$

$$Y_1 = \frac{k^2}{4} \mu \gamma(z)[(x_{01} - x_{02})^2 + (x_{1} - x_{2})^2]$$

$$Y_2 = \frac{k^2}{4} \mu \gamma(z)[(x_{01} - x_{02})^2 + (x_{1} - x_{2})^2].$$

Here $\mu$ and $\gamma$ are random medium parameters defined as follows:

$$\mu = \sqrt{\pi} \frac{R_0 L^3}{\gamma^3}$$

$$\gamma(z) = \frac{2}{(3-n)(2-n)(1-n)} \left( \frac{z}{L} \right)^{3-n} - \frac{n}{1-n} \left( \frac{z}{L} \right)^{2} + \frac{n}{2-n} \left( \frac{z}{L} \right)^{1} - \frac{1}{3} \frac{n}{3-n}.$$
Here $B_0$ is a constant, $L$ is a rough size of the range of the random medium (see figure 1), the positive index $n$ denotes the thickness of the transition layer from the random medium to free space and $n = \frac{k}{4}$ is assumed in section 3 as in [9].

$G_0$ is the Green function in free space. We can obtain the RCS by using equation (12)

$$\sigma = \langle |u_s(r)|^2 \rangle k(4\pi z)^2.$$  

(23)

3. Numerical results

Although the incident wave becomes sufficiently incoherent, we should pay attention to the SCL of the incident wave [11]. The degree of spatial coherence is defined by

$$\Gamma(\rho, z) = \frac{\langle G(r_1|r_t)G^*(r_2|r_t) \rangle}{\langle |G(r_0|r_t)|^2 \rangle}$$

(24)

where $r_1 = (\rho, 0)$, $r_2 = (-\rho, 0)$, $r_0 = (0, 0)$, $r_t = (0, z)$. In the following calculation, we assume $B(r, r) = B_0$ and $kB_0L = 3\pi$; therefore, the coherence attenuation index $\alpha$ defined as $k^2B_0L/4$ given in [10] is $15\pi^2$, $44\pi^2$ and $150\pi^2$ for $kl = 20\pi$, $58\pi$ and $200\pi$, respectively, which means that the incident wave becomes sufficiently incoherent. The SCL is defined as the $2k\rho$ at which $|\Gamma| = e^{-1} \approx 0.37$. Figure 2 shows a relation between SCL and $kl$ in this case and that the SCL = $3$, $5.2$ and $9.7$. We will use the SCL to represent one of the random medium effects on RCS.

Here, we point out that $N$ in (10) depends on the target parameters and polarization of incident waves. For example, we choose $N = 24$ at $\delta = 0.1$ for E-wave incidence in the range of $0.1 < ka < 5$; at $ka = 20$, we choose $N = 40$ at $\delta = 0.1$. As a result, our numerical results are accurate because these values of $N$ lead to convergence of RCS.

Based on the assumption of wave coherence completion in the propagation of distance $2a$, let us define the effective illumination region (EIR) as that surface which is illuminated by the incident wave and restricted by the SCL as shown in figure 3. Therefore, we expect that the target configuration including $\delta$ and $ka$ together with SCL are going to affect the EIR and accordingly the RCS and the enhancement factor of ERCS by a way that will be clarified in section 3. In the following, we conduct numerical results for RCS and normalized radar cross section (NRCS), defined as the ratio of RCS in random media $\sigma$ to RCS in free space $\sigma_0$. 

Figure 2. The degree of spatial coherence of an incident wave about the cylinder.
First we discuss the numerical results for RCS shown in figure 4. We notice from this figure that there are two effects on the RCS. The first is the effect of SCL; as the SCL increases, the behaviour of RCS in random media becomes closer to its behaviour in free space except for the magnitude of RCS. The second is the effect of target curvature and can be seen clearly with changing $\delta$. As $\delta$ increases and/or SCL decreases, as the RCS decreases due to the decrease of EIR, and vice versa. Also, it is observed that RCS suffers from oscillated behaviour and we attribute this manner to the random medium effect.

\subsection{Radar cross section (RCS)}

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3.2. Backscattering enhancement

We consider next the NRCS to manifest the ERCS in random media compared to free space propagation and hence we present numerical results for NRCS in figure 5.

As an extension of our previous work in [11], we analyse the NRCS in three regions of $ka$ compared to SCL.
For $ka \ll SCL$, the NRCS equals two due to the double passage effect of waves in random media. This value of NRCS is realized, independent of illumination portion curvature, i.e. independent of the concavity index $\delta$. This fact is ensured by figure 6 whose NRCS approaches obviously 2.

For $ka \simeq SCL$, the NRCS oscillates largely and irregularly around 2. In fact, this irregular oscillation is due to the difference in the behaviour of RCS, which decreases with $\delta$ as shown in figure 4, between the free space and random medium cases. However, the strength of this oscillation, i.e. enhancement factor, in NRCS decreases with SCL.

For $ka > SCL$, NRCS oscillates regularly in sinusoidal behaviour, with frequency approximately equals $\pi/2$, and descending amplitude with $ka$ approaching certain values, as shown in figures 7–9; this oscillation becomes slight and closer to two with increasing SCL as is ensured in figures 10 and 11 due to EIR widespread. As previously mentioned in section 3.1 this oscillated behaviour is referring to the random medium effect. However, in figures 8 and 9, this descending sinusoidal behaviour appears only in the region of $ka \gg SCL$ because of the gradual weak effect of $\delta$ on the EIR with $ka$. On the other hand, compared to the case of a concave–convex target, for the circular cross section target in figure 7, the sinusoidal behaviour appears earlier because of the zero value of $\delta$ and therefore the EIR depends only on $ka$. In the region $ka \gg SCL$, the impact of $\delta$ and $ka$ become very limited on the EIR and therefore the amount of scattered waves changes very slightly. Accordingly, certain values of NRCS become almost invariable with $\delta$ and $ka$.

There are small differences in the certain values between the typical circular and the concave–convex targets. In figures 7–9, we show these certain values by drawing straight lines. For example, in figures 8 and 9, NRCS $\rightarrow 1.84, 1.93$ in (a) and (b), respectively. However, in figure 7, NRCS $\rightarrow 1.75, 1.9$ in (a) and (b), respectively.
Figure 8. As figure 7, but for $\delta = 0.1$.

Figure 9. As figure 7, but for $\delta = 0.15$. 
4. Conclusion

In this paper we have evaluated the RCS and analysed numerically the behaviour of the ERCS of partially convex targets of large sizes of about five wavelengths in continuous random media. Here, we assume that waves around a target are incoherent and the SCL retains a finite value. In this work, we considered the case when the incident wave illuminates the convex portion of a concave–convex target.

From numerical results, we have found out that there are two important effects on the RCS: the target configuration and the SCL which is more clearly noticed with behaviour of ERCS.

We could also handle ERCS and analyse the behaviour of NRCS in three regions of $ka$ compared to SCL. For $ka \ll SCL$, NRCS equals 2 due to the double passage effect on waves in random media. This value of NRCS holds, independent of the complexity of the target. For $ka \simeq SCL$, NRCS oscillates irregularly and obviously around 2 with target complexity. On the other hand, for $ka > SCL$, NRCS oscillates, approaching certain values that depend on the SCL and almost irrespective of target complexity with $ka > SCL$. Furthermore, these values become closer to 2 with large SCL.

As a result, we can conclude that in the regions $ka \ll SCL$ and $ka > SCL$, the ERCS is almost 2 due to the double passage effect; and in between these two regions ERCS deviates from 2 depending on the target configuration and the SCL.
References