Backscattering from conducting targets in continuous random media for circular polarization

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The polarization of incident waves is one of the key factors in radar detection and remote sensing problems. In a previous study, attention was drawn to the anomalous increase in the radar cross-section (RCS) of a target in a random medium that occurs with H-polarization. This large increase occurs with a small-size target and is attributed to the coupling between the direct and creeping waves. In this work, we aim to probe the effect of the creeping waves on the scattering waves for circular wave polarization and compare it with the previous results. Therefore, we can control the target detection by choosing the proper polarization, which does not lead to anomalous phenomena. In doing so, we present numerical results for RCS and analyse the characteristics of the enhancement in the RCS (ERCS) behaviour of targets in random media. In this regard, we assume partially convex targets of different configurations. We consider the case in which a directly incident wave is produced by a line source distributed uniformly along the axis parallel to the conducting cylinder (target) axis. Then we can deal with this scattering problem two-dimensionally under the condition of strong continuous random media with large local scale size.

1. Introduction

The propagation and scattering of waves in random media have been studied extensively for many years. Radar cross-section (RCS) is one of the key parameters in electromagnetic wave sensing and it becomes an interesting field of research when we consider the case in which the target is surrounded by a random medium and consequently the backscattering enhancement phenomenon will arise [1]. Owing to the correlation of inhomogeneities on the forward and reflected wave paths, an enhancement in the RCS is produced [2–4].

Scattering waves propagating in continuous random media are calculated efficiently by a method that uses a current generator to clarify the medium effects on radar detection [5–11]. This method solves the problem of waves scattered from targets in random media as a boundary value and this solution technique is important for radar detection of a target of finite size. This method uses two operators: the current generator that transforms the incident wave falling on the target into the surface current and Green’s function in the random medium. In earlier investigation [7], numerical results for RCS of conducting convex bodies such as elliptic cylinders have elucidated that the spatial coherence length (SCL) of incident waves around the target is one of the leading parameters for the clarification of random media effects on the RCS.

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Later, we aim to investigate the effect of target configuration on the enhancement phenomenon in RCS (ERCS). In doing that, we have analysed numerically the ERCS of a perfectly conducting cylinder of non-convex cross-section [8, 9]. In those studies we have handled the scattering problem for the condition in which the wavelength and the target size are comparable, and we gave exact results in the small target size. Recently, we considered targets taking large sizes for linear polarizations including E-polarization (E-wave incidence) [10] and H-polarization (H-wave incidence) [11].

In a low-frequency limit, the interference between the direct and creeping waves deep in the shadow region that occurs in the case of H-wave incidence results in a large increase in the backscattering wave intensity, which leads to anomalous ERCS. However, such a phenomenon is absent in the case of E-wave incidence. On the other hand, in the high-frequency limit, only the specularly reflecting points at the surface contribute to the scattered fields and therefore the effect of creeping waves becomes almost negligible.

In this work, we extend our research to involve the study of the effect of circular wave polarization (C-wave incidence) on the RCS and ERCS of targets with different parameters. Therefore we can have a better understanding of the polarization effect in radar engineering. Also we will be able to have some sort of control on target detection by avoiding unwanted phenomena such as resonance that could depend on the polarization of incident waves. In this regard, we consider the case in which the incident wave becomes incoherent around a conducting cylinder although it maintains a finite SCL.

The time factor $\exp(-iwt)$ is assumed and suppressed in the following section.

2. Scattering problem

The geometry of the wave scattering problem is shown in figure 1. A random medium is assumed as a sphere of radius $L$ around a target of the mean size $a \ll L$, and also to be described by the dielectric constant $\varepsilon(r)$, the magnetic permeability $\mu$ and the electric conductivity $\sigma$. For simplicity $\varepsilon(r)$ is expressed as

$$\varepsilon(r) = \varepsilon_0 [1 + \delta \varepsilon(r)]$$

(1)

where $\varepsilon_0$ is assumed to be constant and equal to free space permitivity and $\delta \varepsilon(r)$ is a random function with

$$\langle \delta \varepsilon(r) \rangle = 0, \quad \langle \delta \varepsilon(r) \delta \varepsilon(r') \rangle = B(r, r')$$

(2)

Figure 1. Geometry of the problem of wave scattering from a conducting cylinder in random media.
and
\[ B(r, r') \ll 1, \quad kl(r') \gg 1. \] (3)

Here, the angular brackets denote the ensemble average and \( B(r, r') \), \( l(r) \) are the local intensity and local scale-size of the random medium fluctuation, respectively. Also \( \mu \) and \( \sigma \) are assumed to be constant \( \mu = \mu_0, \sigma = 0 \).

For practical turbulent media the conditions \( B(r, r') \ll 1, kl(r') \gg 1 \) may be satisfied, where \( k = \omega/\sqrt{\varepsilon_0 \mu_0} \) is the wave number in free space. Therefore, we can assume the multi forward scattering and single backscattering approximation and the scalar approximation [12]. Consider the case in which a directly incident wave is produced by a line source \( f(r') \) distributed uniformly along the \( y \)-axis. Then, the incident wave is cylindrical and becomes plane approximately around the target because the line source is very far from the target. Here, let us designate the incident wave by \( u_{in}(r) \), the scattered wave by \( u_s(r) \) and the total wave by \( u(r) = u_{in}(r) + u_s(r) \). The target is assumed to be a conducting cylinder, the cross-section of which is expressed by

\[ r = a[1 - \delta \cos 3(\theta - \phi)] \] (4)

where \( \phi \) is the rotation index and \( \delta \) is the concavity index. We can deal with this scattering problem two-dimensionally using equation (3); therefore, we represent \( r \) as \( r = (x, z) \). Depending on the polarization of incident waves, \( E_y \) or \( H_y \), where \( E_y, H_y \) are the \( y \) components of electric and magnetic fields, respectively, we can impose two types of boundary conditions on wave fields on the cylinder surface \( S \): the Dirichlet condition (DC) for \( E \)-wave incidence and the Neumann condition (NC) for \( H \)-wave incidence

\[ u(r) = 0, \quad \text{for DC} \] (5)
\[ \frac{\partial}{\partial n}u(r) = 0, \quad \text{for NC} \] (6)

where \( \partial/\partial n \) denotes the outward normal derivative at \( r \) on \( S \). In equation (5), \( u(r) \) represents \( E_y \) while in equation (6) \( u(r) \) represents \( H_y \).

According to our method [6–11], using the current generator \( Y_E, Y_H \), and Green’s function in random medium \( G(r \mid r') \), we can express the surface current wave as

\[ J(r_2) = \int_S Y_E(r_2 \mid r_1)u_{in}(r_1 \mid r_1)dr_1 \quad \text{for E-wave incidence} \]
\[ = -\int_S Y_H(r_2 \mid r_1)u_{in}(r_1 \mid r_1)dr_1 \quad \text{for H-wave incidence} \] (7)

where \( u_{in}(r_1 \mid r_1) = G(r_1 \mid r_1) \), whose dimension coefficient is understood. Then, the scattered wave is given by

\[ u_s(r) = \int_S J_E(r_2)G(r \mid r_2)dR_2 \quad \text{for E-wave incidence} \]
\[ = \int_S J_H(r_2)\frac{\partial G(r \mid r_2)}{\partial n_2}dr_2 \quad \text{for H-wave incidence} \] (8)

which can be represented by

\[ u_{se}(r) = -\int_S dr_1\int_S dr_2[G(r \mid r_2)Y_E(r_2 \mid r_1)u_{in}(r_1 \mid r_1)] \quad \text{for E-wave incidence} \] (9)
\[ u_{sh}(r) = -\int_S dr_1\int_S dr_2\left[\left(\frac{\partial}{\partial n_2}G(r \mid r_2)\right)Y_H(r_2 \mid r_1)u_{in}(r_1 \mid r_1)\right] \quad \text{for H-wave incidence} \] (10)
Therefore, the average intensity of the backscattering wave for E-wave incidence is given by

\[
\langle |u_{se}(r)|^2 \rangle = \int_S \text{d}r_0 1 \int_S \text{d}r_0 2 \int_S \text{d}r_1 \int_S \text{d}r_1' Y_E(r_01 \mid r_1') Y_E^*(r_02 \mid r_1') \times \langle G(r \mid r_01)G(r \mid r_02)G^*(r \mid r_1')G^*(r \mid r_1') \rangle
\]

and for H-wave incidence as

\[
\langle |u_{sh}(r)|^2 \rangle = \int_S \text{d}r_0 1 \int_S \text{d}r_0 2 \int_S \text{d}r_1 \int_S \text{d}r_1' Y_H(r_01 \mid r_1') Y_H^*(r_02 \mid r_1') \times \frac{\partial}{\partial n_01} \frac{\partial}{\partial n_02} \langle G(r \mid r_01)G(r \mid r_02)G^*(r \mid r_1')G^*(r \mid r_1') \rangle
\]

Here, \( Y_E \) and \( Y_H \) are the operators that transform incident waves into surface currents on \( S \) and depend only on the scattering body. The current generator can be expressed in terms of wave functions that satisfy the Helmholtz equation and the radiation condition. That is, for E-wave incidence and H-wave incidence, the surface current is obtained as

\[
\int_S Y_E(r_2 \mid r_1) u_{in}(r_1 \mid r_1) \text{d}r_1 \simeq \Phi_M^T(r_2) A_E^{-1} \int_S \ll \Phi_M(r_1), u_{in}(r_1 \mid r_1) \gg \text{d}r_1
\]

\[
- \int_S Y_H(r_2 \mid r_1) u_{in}(r_1 \mid r_1) \text{d}r_1 \simeq - \frac{\partial \Phi_M^T(r_2)}{\partial n} A_H^{-1} \int_S \ll \Phi_M(r_1), u_{in}(r_1 \mid r_1) \gg \text{d}r_1
\]

where

\[
\int_S \ll \Phi_M(r_1), u_{in}(r_1 \mid r_1) \gg \text{d}r_1 \equiv \int_S \phi_m(r_1) \frac{\partial u_{in}(r_1 \mid r_1)}{\partial n} - \frac{\partial \phi_m(r_1)}{\partial n} u_{in}(r_1 \mid r_1) \text{d}r_1
\]

The above equation is sometimes called ‘reaction’, named by Rumsey [13]. Here, the basis functions \( \Phi_M \) are called the ‘modal functions’ and constitute the complete set of wave functions satisfying the Helmholtz equation in free space and the radiation condition \( \Phi_M = \{ \phi_{-N}, \phi_{-N+1}, \ldots, \phi_N \}, \quad M = 2N + 1 \) is the total mode number, \( \phi_m(r) = H_n^{(1)}(kr) \text{exp}(im\theta) \), and \( A_E \) is a positive definite Hermitian matrix given by

\[
A_E = \begin{pmatrix}
(\phi_{-N}, \phi_{-N}) & \cdots & (\phi_{-N}, \phi_{-N}) \\
\vdots & \ddots & \vdots \\
(\phi_N, \phi_{-N}) & \cdots & (\phi_N, \phi_N)
\end{pmatrix}
\]

in which its \( m, n \) element is the inner product of \( \phi_m \) and \( \phi_n \):

\[
(\phi_m, \phi_n) \equiv \int_S \phi_m(r) \phi_n^*(r) \text{d}r
\]

\( A_H \) is \( A_E \) of equation (16) with \( (\phi_m, \phi_n) \) replaced by \( \partial \phi_m / \partial n, \partial \phi_n / \partial n \).

The \( Y_E \) and \( Y_H \) are proved to converge in the sense of mean on the true operator when \( M \rightarrow \infty \). We can obtain the RCS \( \sigma \) by using equations (11) and (12) as follows:

\[
\sigma_e = \langle |u_{se}(r)|^2 \rangle \cdot k(4\pi z)^2 \quad \text{for E-wave incidence}
\]

\[
\sigma_h = \langle |u_{sh}(r)|^2 \rangle \cdot k(4\pi z)^2 \quad \text{for H-wave incidence}
\]

\[
\sigma_c = \sqrt{\sigma_e^2 + \sigma_h^2} \quad \text{for C-wave incidence}
\]

We use equation (20) to calculate RCS for circular polarization since under equation (3) the scalar approximation is valid, as pointed out above. This means that the statistical coupling
between E- and H-polarizations can be neglected and each polarization holds independently in random media.

3. Numerical results

Although the incident wave becomes sufficiently incoherent, we should pay attention to the SCL of the incident wave [6–11]. The degree of spatial coherence is defined by

\[ \Gamma(\rho, z) = \frac{(G(r_1 | r) G^*(r_2 | r))}{(|G(r_0 | r)|^2)} \]  

(21)

where \( r_1 = (\rho, z), r_2 = (-\rho, z), r_0 = (0, 0) \) \( r_i = (\rho, 0) \). In the following calculation, we assume \( B(r, r) = B_0 \) and \( kB_0L = 3\pi \). Therefore the coherence attenuation index (\( \alpha \)) defined as \( k^2 B_0 L / 4 \) and given in [6] is \( 15\pi^2 \), \( 44\pi^2 \), and \( 150\pi^2 \) for \( kl = 20\pi \), \( 58\pi \), and \( 200\pi \), respectively, which means that the incident wave becomes sufficiently incoherent. The SCL is defined as the \( 2k\rho \) at which \( |\Gamma| = e^{-1} \approx 0.37 \). Figure 2 shows a relation between SCL and \( kl \) in this case and that the SCL is equal to 3, 5.2, and 9.7. We will use the SCL to represent one of random medium effects on RCS. Under this condition, we may assume that the fourth-order moment of Green’s functions is expressed as the sum of products of the second-order moment.

Based on the assumption of the waves’ coherence completion in the propagation of distance \( 2a \), let us define the effective illumination region (EIR) as that surface which is illuminated by the incident wave and restricted by the SCL as shown in figure 3. Therefore, we expect that the target configuration including \( \delta \) and \( ka \) together with SCL are going to affect the EIR, and accordingly the RCS and the enhancement factor of ERCS, in a way that will be clarified in the following sections.

In the following, we conduct numerical simulations for RCS and normalized RCS (NRCS) of conducting targets. NRCS is defined as the ratio of RCS in random media \( \sigma \) to RCS in free space \( \sigma_0 \). Numerical results will be analysed with a variety of parameters including the

![Figure 2. The degree of spatial coherence of an incident wave about the cylinder.](image-url)
3.1 Radar cross-section

From the numerical results in figure 4, we notice that there are two factors that obviously affect the RCS. The first factor is the effect of SCL: as the SCL increases, the behaviour of RCS in random media becomes closer to its behaviour in free space except for the magnitude of RCS apart from the configuration of target and incident angle. Moreover, this effect of SCL is realized irrespective of polarization of wave incidence by comparing it with the study on linear polarization in [8–11]. The second factor is the effect of target curvature and can be seen clearly with \( \sigma \) and the angle of incidence. In other words, the RCS with a concave illumination region is larger than that with a convex illumination region due to the widespread EIR. Furthermore, we notice that RCS increases with \( \delta \) in the case of a concave illumination region; however, RCS decreases with \( \delta \) with a convex illumination region. This is also attributed to the effect of EIR that can be explained as follows: in the case of a convex illumination region, \( \delta \) increases as the EIR decreases and that leads to a gradual diminishing of RCS; however, for the concave region case, the EIR increases with \( \delta \) and, accordingly, the RCS increases.

The behaviour of RCS undergoes smooth fluctuations in both cases of free space and random media. However, this oscillated behaviour is less than that in the case of H-wave incidence as already shown in previous work [11]. For the free-space case, the oscillated behaviour, is produced by the effect of creeping waves [14]. At the point of tangency, some of the incident wave creeps around the surface at a velocity less than that in free space and that is attenuated by tangential radiation. For low \( ka \) values, the waves can continue to creep around the target many times and, therefore, the interference between the specularly reflected and creeping waves is obvious enough to affect the RCS, resulting in oscillated behaviour. However, with larger \( ka \) values, the creeping waves travel along the cylinder and due to radiation, they become weaker and weaker the further they have to travel. Therefore the creeping waves attenuation reduces its effectiveness rapidly resulting in diminishing the interference effect gradually with \( ka \). For the random medium case, the oscillation is attributed to the effect of creeping waves propagating in a random medium that in turn has another effect on the RCS, as has been pointed out in our papers [10, 11].

3.2 Backscattering enhancement

Here we numerically analyse ERCS by showing results, of our simulator, for NRCS in figure 5. We note clearly an important observation that characterizes wave scattering from partially convex targets in random media. That is the difference in the behaviour of the NRCS between
both concave and convex illumination regions as a result of the EIR effect, as explained previously. Owing to the double passage effect [1], when $SCL \gg ka$, NRCS equals two. This fact holds also with linear polarization of the incident waves, as shown in [8–11]. However, NRCS deviates from this value with increasing $ka$ and/or $\delta$; this deviation in NRCS decreases with the increase in SCL. For the concave illumination area, NRCS decreases gradually with some fluctuations as a result of the difference in the creeping waves effects between both
Figure 5. Normalized RCS versus target size at two different incident angles where (a) $\delta = 0$, (b) $\delta = 0.1$, (c) $\delta = 0.2$.

free space and random media. On the other hand, for the convex illumination area, NRCS fluctuates slowly close to two; this fluctuation increases with $\delta$. This analysis demonstrates clearly the effect of the illumination region curvature in conjunction with SCL of incident waves on NRCS.

As shown in [8, 9], NRCS suffers from anomalous increase with H-wave incidence; however, this large increase is absent with the C-wave incidence. This is due to the decrease in the
creeping wave’s effect on the RCS with C-wave incidence. This observation clarifies that the resonance phenomenon occurs only with H-wave incidence for the low-frequency range.

4. Conclusion

In this paper, we have studied the scattering problems of plane waves incidence on targets in free space and random media for circular polarization. The RCS of a target with a concave–convex cross-section was calculated for different incident angles and SCL.

We have altered the effects that have a clear influence on the RCS and backscattering enhancement, they are: the target configuration and the SCL that is more clearly noticed with the behaviour of the enhancement in RCS. These characteristics are realized apart from the incident angle and the circular polarization, as well as with linear polarizations. The behaviour of RCS undergoes smooth fluctuations in both free space and random media. However this oscillated behaviour is less than that in the case of H-wave incidence, as already shown in [11].

In conclusion, we have shown that RCS behaviour depends largely on the polarization of incident waves. Therefore, we stress that one should avoid using H-wave incidence to detect targets with low-frequency limits and use instead E-wave or C-wave incidences.

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