The Cost Channel in the New Keynesian Model: Comparing
Inflation Targeting and Price-level Targeting

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Preliminary Draft
November 2003

Abstract

A standard result in a typical new Keynesian model is that a central bank faces a trade-off between stabilizing actual output around the level of output under flexible prices and stabilizing the inflation rate only in the presence of cost-push shocks and that demand shocks are completely offset. This paper develops a model in which the bank lending channel or the cost channel of monetary policy transmission (a cost channel arises when firms’ marginal cost depends directly on the nominal interest rate) is introduced in an otherwise standard new Keynesian model and explores its implications for monetary policy trade-offs under optimal discretionary and optimal pre-commitment inflation targeting regime. An important result is that the central bank now faces a trade-off between stabilizing output-gap and inflation in the presence of demand shocks as well. Moreover, the gains from commitment are larger in the presence of a cost channel. The model also studies the properties of a price-level targeting regime and shows that it out-performs the inflation targeting case by reducing the expected losses of the central bank.

JEL Classification: E52, E58, E61

Keywords: Optimal monetary policy, inflation targeting, price-level targeting, bank-lending channel.
1- Introduction

Academic thinking about monetary economics has changed drastically over the past decade and so has the practice of monetary policy. Almost simultaneously, big advances were made in the management of monetary policy on the one hand and in theoretical and empirical research on monetary policy on the other hand. The theoretical developments have culminated in a simple framework that is based on intertemporal optimizing behaviour, rational expectations, and temporary price rigidities and in which monetary policy have both short-term or temporary output effects and long-term or permanent price effects. This new model is generally referred to as the ‘New Neoclassical Synthesis model’ or the ‘New Keynesian’ model and is widely used to assess the desirability of alternative monetary policies.¹

Although this new framework has come a long way in integrating the ideas of New Keynesians with those of New Classicals,² it still lacks in its treatment of monetary policy transmission mechanism. The objective of this paper is to fill this gap in the literature by developing a model in the tradition of new Keynesian models that incorporate two alternative and independent channels of monetary policy transmission mechanism. The model is then used to analyze two popular monetary policy targeting regimes --- inflation targeting and price-level targeting.

¹ For an excellent survey of recent developments and results in this area, see Gali (2002).
² Typically, New Keynesians focus on market imperfections and nominal rigidities (such as price stickiness) to generate real effects of monetary policy while new Classical emphasize on intertemporal optimization and micro-foundations for the aggregate macroeconomic behavior based on a flexible price dynamic general equilibrium model.
In a typical new Keynesian model, the traditional interest rate channel is used to explain the effects of monetary policy. According to this channel, a change in the interest rate affects the spending decisions of households and firms and thus operates only through the aggregate demand side of the model. An alternative channel, that has received considerable attention in recent years, emphasizes the existence of frictions (such as limited participation) and information asymmetries (that lead to the problem of adverse selection and moral hazard) in the financial markets as an important source of monetary non-neutralities. This channel is usually termed as the credit channel and has the potential to affect the spending decision of firms by affecting their net worth --- the financial accelerator mechanism, and their production decisions by affecting their cost of production --- the bank lending channel. Thus, the broad credit channel can have an effect on the demand side and the supply side of the model. Until recently, the credit channel was mostly studied in flexible price models. For example, Christiano, Eichenbaum and Evans, in a number of important papers have introduced the bank lending channel in a flexible-price dynamic general equilibrium model\(^3\). Bernanke, Gertler and Gilchrist (1999) is an excellent example where the financial accelerator mechanism is incorporated in a sticky-price general equilibrium model. However, the bank lending channel, that affects the supply side of the model, has received very little attention in a simple new Keynesian model. \(^4\)

\(^3\) See Christiano and Eichenbaum (1992) and Christiano, Eichenbaum and Evans (1997).

\(^4\) Two exceptions that I have come across in this regard are Christiano, Eichenbaum and Evans (2001) and Wlash (2003).
This paper incorporates the bank lending channel or simply the cost channel of monetary policy (because the bank lending channel affects the cost of production of firms) in an otherwise standard new Keynesian model. By analyzing both the traditional and the cost channel of monetary policy in one unified framework, this paper is an attempt to bridge the gap between these two parallel strands of literature. The model is then used to provide new insights on two well-known questions in the literature on monetary policy. First, what should be the ultimate goal of monetary policy and which variable(s) should the central bank target to achieve this goal? Second, should the central bank commit to rules or follow discretion in achieving those targets? In recent years, a consensus has emerged that central banks should primarily focus on policies that promote long-term price stability in the economy, and that there are gains from credibly committing to a rule-based monetary policy regime. However, more research needs to be done on the properties of specific intermediate targeting regimes and the feasibility and implications of commitment. As Clarida, Gali and Gertler (1999) conclude, “though substantial progress has been made, our understanding of the full practical implications of commitment for policy-making is still at a relatively primitive stage, with plenty of territory that is worth exploring”. This paper is one step forward in that direction.

Distinguishing the relative importance of the traditional and the cost channel is useful for various reasons. First, it improves our understanding of the link between the financial and real sectors of the economy. Second, it provides alternative indicators to help gauge the stance of monetary policy and thus increases its ability to offset particular types of adverse shocks. Third, a clear understanding of the transmission mechanism has the

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5 For a detailed discussion see Kashyap and Stein (1994)
potential to give more information regarding the choice of intermediate targets. Moreover, several researchers like Christiano and Eichenbaum (1992), Christiano, Eichenbaum and Evans (1997, 2001) and Barth and Ramey (2001) have emphasized on the cost channel as a powerful collaborator of the traditional channel in the transmission of short run effects of monetary policy. Building on these observations the paper demonstrates two results for an inflation targeting regime. First, the central bank faces a trade-off between stabilizing inflation and output-gap in the presence of both demand and cost-push shocks. This result is different from the one reported by Clarida, Gali and Gertler (1999) in a standard new Keynesian model. They argue that the central bank is able to perfectly off-set the demand shock and only faces an inflation and output-gap volatility tradeoff in the presence of cost-push shock. However, when the cost channel is operating, an increase in the nominal interest rate to counter the effects of a positive demand shock not only reduces the output-gap thus reducing inflation but also increases inflation directly. Thus, the central bank is better off in trading some volatility in the output-gap for reduced volatility in inflation. Second, the presence of cost channel increases the gains from commitment. Moreover, the outcome of optimal commitment monetary policy is superior compared to optimal discretionary policy even in the absence of a classic inflation bias and even in the absence of persistence in the shock processes. Emphasizing on the importance of forward-looking behaviour, the standard new

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6 By inflation targeting, I mean when the central bank attempts to stabilize both output-gap and inflation. Svensson (1997) calls this ‘flexible’ inflation targeting.

7 The classic literature, in the tradition of Barro and Gordon (1983), on rules versus discretion and the role of credibility in monetary policy maintain that if the central bank has a desire to push the economy’s output level beyond the natural rate then discretion would only lead to higher average inflation (thus the term inflation bias) with no effect on output; credibly committing to a rule would eliminate the inflation bias.
Keynesian model predicts that the central bank only gains from commitment when the cost-push shock exhibits some persistence (even if there is no inflation bias).

The paper extends the analysis further by seeking an answer to the question: If (for practical reasons) it is difficult to credibly commit to a monetary policy rule, is it possible to delegate the central bank an alternative discretionary targeting regime (a different loss function) that would replicate the commitment solution? Woodford (1999) argues that this is possible if the central bank is assigned a loss function with interest rate as an explicit argument. Jensen (2002) has come-up with a similar answer in which the central bank targets nominal income growth rate. Vestin (2000) has also reached a similar conclusion by considering a price-level targeting regime. The intuitive explanation for this conclusion is the same in all these papers; they imply inertial behaviour in the discretionary monetary policy that is a feature of the commitment solution. It is the price-level targeting regime that I consider as an example to highlight the importance of cost channel of monetary policy to verify the above mentioned claims. Before discussing the results, let me briefly highlight the differences between inflation targeting and price-level targeting and what the literature says on their relative merits.

Although price-level targeting is quite similar to inflation targeting (that has become extremely popular in recent years in many developed countries; see Bernanke and Mishkin (1997) for details)) and share many of its benefits, the two regimes have a fundamental difference. Under inflation targeting, the monetary authority let bygones be bygones while under price level targeting they attempt to remedy the past failures. More
specifically, if there is an unexpected increase in prices then according to price level targeting the monetary authority will attempt to deflate prices back to the original in order to prevent the base drift in the price level, while under inflation targeting no action will be taken and the new level of prices would be maintained. This means that the long-term price level is more certain under price level targeting while it may wander around randomly over long periods under inflation targeting. However, short-term price volatility (and thus output volatility) may be higher under price-level targeting because unexpected rises in the price level will be followed by attempted reductions in the price level.

The conventional literature like Fischer (1994) and Haldane and Salmon (1995) focus on this alleged increased output-gap volatility under price-level targeting to argue against it. Kiley (1998) has also reached a similar conclusion using a new Keynesian model. However, Dittmar, Gavin and Kydland (1999) and Svensson (1999) have challenged this conventional wisdom and, employing a neo-classical Phillips curve, shown the price-level targeting to be preferred over inflation targeting. More recently, Dittmar and Gavin (2000) and Vestin (2000) have confirmed this result using the new Keynesian Phillips curve by demonstrating that price-level targeting provides a better inflation-output-gap variability trade-off compared to inflation targeting with discretionary policy making. Vestin (2000) takes this line of reasoning a step further and proposes that the price-level target replicates the commitment solution of inflation targeting when there is no persistence in the cost-push shock.
Thus, the debate over the relative benefits of inflation targeting and price-level targeting is far from being settled.8 As Mishkin (2001) has correctly pointed out that the results in favour of or against a price level target are very model specific, especially regarding the specification of the Phillips curve. In particular, the assumptions about private sector’s inflation expectations entering the Phillips curve, amount of persistence in the output gap and whether policy is conducted under a commitment rule or in a discretionary fashion play important roles in determining the desirability of price level targeting. In this paper, I add one more consideration; namely, the cost channel of monetary transmission and demonstrate that the price-level targeting is preferable to inflation targeting as it lowers the expected value of the loss incurred by the central bank. However, this comparison is conducted after appropriately adjusting the relative weight the central bank places on output-gap stabilization and inflation stabilization.

2- The Model

The model developed here introduces supply-side effects of interest rates or the cost channel of monetary policy in an otherwise standard New Keynesian model widely used for analysis of monetary policy. The basic framework employed is a variant of a cash-in-advance model with sticky prices. There are four types of economic agents in the economy: households, firms, the monetary authority and financial intermediaries or simply banks. Given their preferences, households decide how much to consume the differentiated composite consumption good, how to allocate time between leisure and

8 For an in-depth analysis of the conditions under which price level targeting would be preferred over inflation targeting see Barnett and Engineer (2001).
work --- the labour supply decision, and how much of their money holdings and wage earnings should they deposit with the bank. The firms operate in a monopolistically competitive environment and thus command some monopoly power. They also take three decisions: how much of the differentiated good should they produce using the labour services of the households, how much loan to take from the bank to pay the wages of the hired workers and how to set the price for their output. Each firm sets the price of the good it produces, but not all firms reset their price in each period. The monetary authority issues money and employs nominal interest rates as an instrument of monetary policy to achieve certain well-specified goals. The role of the banks is quite trivial. They receive deposits from the households and a cash injection from the monetary authority and lend this amount to the firms at an interest rate set by the monetary authority.

2.1 - Households

The economy consists of a continuum of identical households indexed by \( i \in [0,1] \). The model will be described in terms of a representative household making decisions in the presence of uncertainties about the future. A typical household seeks to maximize the expected present discounted value of utility:

\[
U(C_t, N_t) = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-1/\sigma}}{1-1/\sigma} - \psi \frac{N_t^{1+\phi}}{1 + \phi} \right]
\]  

(1)

where \( 0 < \beta < 1 \) is the discounted rate of time preference, \( '\sigma' \) represents elasticity of intertemporal substitution, \( '\phi' \) is the elasticity of labour supply and \( E_0 \) denotes the expectation based on the information set available at time zero. \( C_t \) is the composite consumption good, \( N_t \) denotes labour supply, and \( P_t \) is the price index for \( C_t \).
The composite consumption index, $C_t$, consists of differentiated goods produced by firms operating in a monopolistically competitive environment. I assume that consumption is differentiated at the individual goods level. Thus, the goods consumption index can be written as a CES aggregator of the quantities consumed of each type of good.

$$C_t = \left( \int_0^1 C_t(i) \frac{\varepsilon-1}{\varepsilon} di \right)^{\frac{\varepsilon}{\varepsilon-1}}$$

(2)

where the parameter ‘$\varepsilon$’ is the elasticity of substitution within each category and is assumed to be greater than one, i.e., $\varepsilon > 1$.

The demand functions for goods within each category can be determined by maximizing equation (2) with respect to the total expenditure on that good, given as $Z_t = \int_0^1 P_t(i)C_t(i)di$, where $P_t(i)$ is the price of the consumption good $C_t(i)$. The demand function that emerges from this maximization exercise is given as:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t$$

(3)

where $P_t$ is the aggregate price index for composite good $C_t$ that satisfy the expenditure equation expressed as $Z_t = P_tC_t$. The expression for $P_t$ is given as:

$$P_t = \left( \int_0^1 P_t(i)^{1-\varepsilon} \right)^{\frac{1}{1-\varepsilon}}$$

(4)

In maximizing utility given by (1), the representative household faces two types of constraints --- cash-in-advance constraint and intertemporal budget constraint. She enters
period $t$ with some initial cash holding, $M_t$, and receives her wage income, $W_tN_t$, as cash. From this total cash amount ($M_t + W_tN_t$) she deposits amount $D_t$ in a bank and uses the remaining amount to purchase consumption goods, $P_tC_t$. Thus, her cash-in-advance constraint can be written as:

$$M_t + W_tN_t - D_t \geq P_tC_t$$  \hspace{1cm} (5)

At the end of the period $t$, she receives from the bank the principal amount she deposited $D_t$, interest accrued on this amount $i_tD_t$, and her share of profit $\Pi^b$. Moreover, she also receives the share of profits $\Pi^f$ from the firms. The remaining amount is carried as cash over to the next period, $t+1$. Thus, the intertemporal budget constraint faced by the representative households can be written as:

$$M_t + W_tN_t - D_t - P_tC_t + (1 + i_t)D_t + \Pi^b + \Pi^f = M_{t+1}$$  \hspace{1cm} (6)

By maximizing (1) subject to the constraints (5) and (6), I can drive the following set of first order optimality conditions:

$$\frac{C_t^{-\frac{1}{\sigma}}}{P_t} = \beta(1 + i_t)E_t \frac{C_{t+1}^{-\frac{1}{\sigma}}}{P_{t+1}}$$  \hspace{1cm} (7)

$$\psi C_t^{\frac{1}{\sigma}N_t} = \frac{W_t}{P_t}$$  \hspace{1cm} (8)

Equation (7) is the standard Euler equation for the optimal intertemporal allocation of consumption. It has the usual interpretation that at a utility maximum, the household cannot gain from feasible shifts of consumption between period $t$ and $t+1$. Equation (8) is the intratemporal optimality condition representing the labour supply decision.

Moreover, the cash-in-advance constraint will be binding in equilibrium:
\[ M_i + W_i N_i - D_i = P_i C_i \] (9)

2.2)- Firms

This section outlines the mechanics of monopolistic competition in a dynamic general equilibrium setting. Like every firm operating in a monopolistically competitive market, each firm in this model has to take two types of decisions --- how much output to produce and at what prices to sell this output that would maximize profits. In doing so, a representative firm ‘i’ is subject to a number of constraints. First is the specification of the production function. Following McCallum and Nelson (1999), I assume that there is no capital in the economy and so the firm only employs the labour input supplied by households to produce the differentiated consumption good consumed by the household:

\[ C_i(i) = A_i N_i(i) \] (10)

where \( A_i = \exp(z_i) \) and ‘\( z_i \)’ represents aggregate technology shock.

The second constraint is the demand function for these differentiated goods, which is given by equation (3). The third constraint introduces price stickiness by assuming that each period some firms are unable to adjust their price. This staggered price adjustment behaviour is based on Calvo (1983). Firms are assumed to face a constant probability \( 1 - \rho \) in every period to alter their price in an optimal fashion. This probability is independent of how long their prices has been fixed and the expected duration of price stickiness is \( 1/\rho \). It is easy to verify that with a large number of firms in the economy, the fraction of firms adjusting price optimally in a period is equal to the probability of
price adjustment $1 - \rho$; the remaining fraction of firms ‘$\rho$’ do not adjust their price. Thus, the parameter ‘$\rho$’ captures the degree of nominal price rigidity.

The fourth constraint is used to incorporate the cost channel of monetary policy. I assume that the representative differentiated-good-producing firm needs to pay the hired workers before receiving the revenues from the sale of output produced. For this purpose, she will borrow an amount $W_iN_i(i)$ from the bank at the interest rate $i_t$ to finance the wage bill.

For analytical simplicity, I assume initially that all firms are able to adjust their prices every period, that is, the third constraint is not binding yet. Then, the profit function for a representative firm ‘$i$’ can be written as:

$$\pi_i(i) = P_i(i)C_i(i) - W_iN_i(i) - i_tW_iN_i(i)$$

(11)

The differentiated-good-producing firm chooses $P_i(i)$ and $N_i(i)$ to maximize these profits subject to the conditional demand for their variety of output given by equation (3) and the production function given by equation (10). The expressions for $P_i(i)$ and $N_i(i)$ are given respectively as:

$$P_i(i) = \frac{\varepsilon}{\varepsilon - 1}MC_i$$

(12)

and

$$\frac{W_i(1 + i_t)}{P_i(i)} = \frac{\varepsilon - 1}{\varepsilon}F_N$$

(13)
where \( \left( \frac{\epsilon}{\epsilon - 1} \right) = \nu \) is the constant mark-up and \( MC_i \) is the minimized nominal marginal cost. \( F_N \) is the marginal product of labour, which, given the production function is simply \( A_i \).

Note that equation (12) just depicts the relationship between the ‘flexible’ price chosen by all firms and the minimized marginal cost of production under monopolistic competition; it does not say anything about prices being sticky. Combining equation (12) and (13) I can write:

\[
MC_i = \frac{W_i(1 + i_i)}{A_i}
\]  

or in real terms as:

\[
mc_i = \frac{W_i(1 + i_i)}{\bar{P}A_i}
\]

Note that, combining equation (13) --- equilibrium labour demand with equation (8) --- equilibrium labour supply, using equation (10) --- the production function, and the goods market equilibrium condition \( Y_t = C_t \), I can derive the equilibrium level of output produced in the economy. This equilibrium level of output represents the flexible-price equilibrium of the economy since I have not introduced sticky prices yet in the model and is given as:

\[
Y_t' = \left[ \frac{1}{\psi} \frac{\epsilon - 1}{\epsilon} \frac{A_i^{1-\phi}}{1 + i_t'} \right]^{\frac{\sigma}{1 + \sigma \phi}}
\]
In addition to technology, the flexible-price output level depends on the mark-up (due to the presence of monopolistic competition) and the nominal interest rate (which represents monetary policy). Thus, even if the distortion created by the monopolistic competition is eliminated, \( Y^f \) will not be efficient as long as \( \delta_t > 0 \).

Now, I introduce price stickiness by assuming that price adjustment does not take place simultaneously for all firms. Following Rotemberg (1987), suppose that a representative firm ‘\( i \)’ that is allowed to change its price, set its price to minimize the expected present discounted value of deviations between the price it sets and the minimized nominal marginal cost.

\[
\sum_{j=0}^{\infty} \rho^j \beta^j E_t (P_t(i) - MC_{t+j})^2
\]  

(17)

where \( MC_t \) is the minimized nominal marginal cost. Note that there are two parts to discounting. The first, \( \beta \) represents a conventional discount factor, and the second, \( \rho \) reflects the fact that the firm that has not adjusted its price after ‘\( j \)’ periods, still has the same price in period \( t+j \) that she set in period \( t \). The first order condition with respect to \( P_t(i) \) gives the following optimal value denoted by \( P_t^*(i) \).\(^9\)

\[
P_t^* = (1 - \rho \beta)MC_t + \rho \beta E_t P_{t+1}^*
\]

(18)

Thus, the optimally chosen price in period \( t \) is a weighted average of nominal marginal cost and expected value of optimal price in the future. However, in period \( t \), only a

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\(^9\) It is reasonable to set \( P_t^*(i) = P_t^* \) because all firms are identical except for the timing of their price adjustment.
fraction $1 - \rho$ of firms set their price according to equation (18). The remaining firms are stuck with the prices set in previous periods. Since the fraction of firms that are able to optimally set their price is randomly chosen, the average price of the previous period will be the price of the fraction of firms that are unable to adjust their price this period. Therefore, the overall aggregate price level in period $t$ is a weighted average of current optimally chosen and past prices. This can be written as:

$$P_t = [\rho^{1-\varepsilon}P_{t-1} + (1 - \rho)^{1-\varepsilon}P^*_t]^{1/\varepsilon} \quad (19)$$

where $P^*_t$ is the price chosen by all adjusting domestic firms in period $t$.

2.3)- Banks

Banks operate costlessly in a competitive environment and play a trivial role in this model. They receive deposits, $D_t$ from the households and lump sum cash injection, $X_t$ from the monetary authority. This amount is supplied/lent to firms at the nominal interest rate, $i_t$. The demand for these loans comes from the firms who need to finance their wage bill, $W_tN_t$. Thus, equilibrium in the loan market requires that:

$$W_tN_t = D_t + X_t \quad (20)$$

The bank pays $(1 + i_t)D_t$ to households in return for their deposits and distributes $(1 + i_t)X_t$ to households in the form of profits.
3 – Log-linearized Model

In this section, the model is log-linearized around the steady state. A variable in lower case represent the log deviation with respect to the steady state. In equilibrium firms are assumed to be symmetric and taking identical decisions. This implies that prices are equal for each variety of good and is equal to the price index given by equation (4). That is, $P_i(i) = P_i$. Also, $N_i(i) = N_i$ and $C_i(i) = C_i$.

3.1 – Goods Market Equilibrium --- the new IS-curve

The log-linearized version of the resource constraint of this economy, $Y_i = C_i$ can be written as:

$$y_t = c_t$$ (21)

In order to derive an IS-type relationship that relates output level the real interest rate, I need to make use of the Euler equation for consumption. The log-linearized version of this relationship is:

$$c_t = E_t c_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + s_t$$ (22)

Note that in deriving equation (22), I have included an additive disturbance term $s_t$ that represents uncertainty (see, McCallum and Nelson (1999)). It can also be justified on the grounds that some non-linear terms are ignored while linearizing the Euler equation for consumption. The disturbance term $s_t$ could also include the taste shocks had I introduced them in the specification of the utility function or it could represent the change in the government spending if it was not assumed to be zero. My objective is not to pin-
down the source of this shock rather to have some disturbance term that represents the demand shocks.

After substituting equations (22) in equation (21), I get a relationship that represents equilibrium in the goods market --- the new IS equation:

\[ y_t = E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) + s_t \]  

(23)

Following the tradition in the recent literature on monetary policy, let \( x_t = y_t - y_t^f \) be defined as the output gap, where \( y_t^f \) is defined as the level of output that arises with perfectly flexible prices. Similarly, let \( r_t^f \) denote the real interest rate that arise in the frictionless equilibrium. Then, I can write equation (23) as:

\[ x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1} - r_t^f) + s_t \]  

(24)

where \( r_t^f \) is defined as follows:

\[ r_t^f = \left( \frac{1}{\sigma} \right) E_t (y_{t+1}^f - y_t^f) + \left( \frac{1}{\sigma} \right) s_t \]  

(25)

\( y_t^f \) can be calculated by log-linearizing equation (16):

\[ y_t^f = \frac{\sigma}{1 + \phi} \left[ (1 + \phi) z_t - i_t^f \right] \]  

(26)

### 3.2 – Inflation Adjustment Behaviour --- the new Phillips curve

The log-linearized version of equation (18) and equation (19) can be combined to produces the following Phillips curve type relationship:
\[
\pi_t = \beta E_t \pi_{t+1} + \gamma mc_t,
\]
where \(\gamma = \frac{(1-\rho)(1-\rho\beta)}{\rho}\)

The expression for \(mc_t\) (real marginal cost) can be had by log-linearizing equation (15).

\[
mc_t = w_t - p_t + i_t - z_t
\]

Using the log-linearized version of the labour supply equation (equation 8) to eliminate \(w_t - p_t\), and using equation (21) to replace \(c_t\) with \(y_t\) and the production function to eliminate \(n_t\), I can rewrite the equation for \(mc_t\) as:

\[
mc_t = \left(1 + \frac{\sigma \phi}{\sigma}\right)y_t - (1 + \phi)z_t + i_t
\]

Note that by setting, \(mc_t = 0\) I get the same expression for \(y_t^f\) as given in equation (26). Thus, by subtracting equation (26) from equation (29), \(mc_t\) can be expressed in output gap form:

\[
mc_t = \left(1 + \frac{\sigma \phi}{\sigma}\right)x_t + (i_t - i_t^f)
\]

Thus, the inflation adjustment behaviour defined in equation (27) can be written as:

\[
\pi_t = \beta E_t \pi_{t+1} + \gamma \left(1 + \frac{\sigma \phi}{\sigma}\right)x_t + \gamma(i_t - i_t^f)
\]

Consider three ad-hoc modifications to this optimally derived Phillips curve. First, consistent with the current literature on monetary policy, I will introduce an ad-hoc cost-push shock, \(\nu_t\) to study the trade-off between stabilizing output-gap variability and inflation variability faced by the monetary authority. In the standard new Keynesian model (without the cost channel), this supply-side shock is necessary to generate a
meaningful monetary policy problem because the demand-side shocks are completely stabilized. However, in this paper it is not necessary to introduce this cost-push shock because demand shocks are not completely stabilized. It would be useful for comparing the results with the baseline case. Second, a new parameter \( \kappa \) will be introduced. By setting its value equal to 0 or 1, I can study the properties of the model when the cost channel is closed and when it is operational. Third, the nominal interest rate under flexible-price equilibrium, \( i^f \) will be set equal to zero. This is done for the sake of simplicity. With flexible prices, the goods market will clear automatically and the monetary authority does not need to alter its policy instrument --- the nominal interest rate --- to stabilize output. In this sense, the nominal interest rate can be treated as a constant and normalized to zero. Thus, a change in monetary policy instrument can be thought of as a deviation from the level of interest rate that prevailed under flexible prices. The resulting Phillips curve is:

\[
\pi_t = \beta E_t \pi_{t+1} + \lambda x_t + \gamma \kappa \hat{v}_t + v_t
\]

where \( \lambda = \gamma \left( \frac{1 + \sigma \phi}{\sigma} \right) \).

### 3.3- The complete model

\[ x_t = E_t x_{t+1} - \sigma (i_t - E_t \pi_{t+1}) + u_t \tag{33} \]

\[ \pi_t = \beta E_t \pi_{t+1} + \lambda x_t + \gamma \kappa \hat{v}_t + v_t \tag{34} \]

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10 Equation (33) is obtained after eliminating \( r_t^f \) from equation (24) using equation (25) and (26). To be precise \( u_t = \frac{1 + \phi}{1 + \sigma \phi}(E_t(z_{t+1} - z_t)) + \frac{1}{\sigma} s_t \), and as mentioned above it could conceivable include taste and government spending shocks.
\[ u_t = \rho_u u_{t-1} + \varepsilon_u^u \tag{35} \]

\[ v_t = \rho_v v_{t-1} + \varepsilon_v^v \tag{36} \]

The key difference between this model and the standard new Keynesian model is the presence of nominal interest rate in equation (34) --- the Phillips curve or the inflation adjustment equation. The reason for its inclusion is the assumption that firms borrow cash from banks to finance their wage bills. Thus, a change in the interest rate directly affects the costs faced by firms and thus their price adjustment behaviour. Since interest rate is the instrument of monetary policy, its inclusion can be termed as the cost channel of monetary policy. Therefore, in addition to the traditional demand-side channel of monetary policy the model also captures the supply-side effects of monetary policy. With \( \kappa = 0 \), the cost channel can be closed and the model then becomes the standard new Keynesian model. The next section analyzes the implications of the cost channel for optimal monetary policy under commitment and discretion. It also highlights the significance of the cost channel while comparing the performance of two alternative monetary policy regimes --- Inflation targeting and Price-level targeting.

### 4)- Optimal Monetary Policy

This section describes the behaviour of the monetary authority --- the central bank. In simple words, the central bank uses its policy instrument (the nominal interest rate) together with the knowledge of the economy as represented by the IS curve (depicting the behaviour of aggregate output) and the Phillips curve (describing the behaviour of aggregate output) and the Phillips curve (describing the behaviour of aggregate output).

---

11 By optimal monetary policy I mean that, given the dynamic general equilibrium structure, the effects of all sources of sub-optimality like nominal price rigidities are fully neutralized and the efficient flexible price equilibrium allocation is restored.
Inflation (and prices)) to achieve certain well-specified goals represented by an objective function (also termed as the loss function) that explicitly translates the behaviour of the target/goal variables into a welfare measure. The objective function thus serves as a guide for the central banks to formulate monetary policy. For a new Keynesian model, Woodford (2002) has derived this monetary policy objective function by taking a second-order linear approximation to the utility function of the representative agent. According to Woodford’s specification of the loss function, the central bank seeks to minimize the discounted sum of squared deviations of output-gap and inflation from their target paths.

\[ L_t = \frac{1}{2} E_t \left[ \sum_{t=0}^{\infty} \beta^t (\alpha \bar{y}_{t+1} + \pi_{t+1}^2) \right] \quad (37) \]

where ‘\( \alpha \)’ is the weight that the central bank places on stabilizing output gap relative to inflation. As Woodford has shown ‘\( \alpha \)’ depends on the deep parameters of the model.

Although equation (37) is very similar to the standard quadratic loss function used extensively in the classic time-consistency literature (pioneered by Kydland and Presscott (1977) and Barro and Gordon (1983)) and in most recent papers (for example, Clarida, Gali and Gertler (1999)) analyzing the properties of basic new Keynesian model, two differences are worth noting. First, the output gap is defined as the difference in the actual aggregate output level and the equilibrium output under flexible prices rather as output level relative to the natural rate of output. The natural rate output typically depends only on technology shocks and the production function parameters. However, as evident from expression (26), the output under flexible prices depends on the utility

\[ 12 \text{ Rather than deriving the objective function for the model in this paper, I will intuitively explain the differences that may arise due to the presence of cost channel and how I assume away from those differences.} \]
function parameters as well. Thus, for example, households labour supply decision will have an impact on the level of output under flexible prices. In addition, in the present model it also depends on the instrument of monetary policy, the nominal interest rate, due to the presence of the cost channel. This can have important implications for policy. It may not be optimal anymore to minimize the gap between actual output level and the output level under flexible prices. This leads to the next point.

Second, it is assumed that the central bank’s target for output is equal to the economy’s equilibrium level of output under flexible prices, that is, a target of zero output-gap $x_t = 0$. Unlike the traditional analysis, this assumption essentially ensures that the central bank has no incentive to increase the economy’s output beyond this level. This seems a little odd in the presence of monopolistic competitive firms. However, consistent with the literature, I have assumed that government fully offsets this market power distortion by subsidizing employment, which is financed through lump sum tax on households. Accordingly, it would be optimal for the central bank to eliminate the effects of nominal price rigidities and attain the flexible price equilibrium. This reasoning would hold in a model without the cost channel. However, with the cost channel present, setting $x_t = 0$ may not be consistent with keeping inflation around zero because targeting $x_t$ at zero requires a change in the interest rate which will cause inflation to move as well. Another way to understand this point is to consider equation (30) --- $mc_t = \left(1 + \frac{\sigma \phi}{\sigma}\right)x_t + (i_t - i^*_t)$. In the absence of the cost channel, setting $mc_t = 0$ would guarantee $x_t = 0$. With the cost channel operating, achieving $x_t = 0$ would require setting $i_t = i^*_t$. But, $i^*_t$ is not
independent of the output level under flexible prices --- \( y^f \). In order to get around this problem, I need to define the concept of output-gap with care. I define it as the gap between actual output level and the output level that would prevail under flexible prices and constant nominal interest rate. As explained above, this constant nominal interest rate with flexible prices is normalized to be equal to one.

A well-known result in the classic literature on optimal discretionary and commitment monetary policy is that if the central bank has no incentive to increase the economy’s level of output beyond the equilibrium flexible-price level of output then there would be no inflation bias problem and the gains from commitment (in terms of eliminating this bias) would essentially be zero. However, Clarida, Gali and Gertler (1999) have argued that in a forward-looking model, such as the basic new Keynesian model, there still may be gains from credible commitment. The reason is that in a forward-looking model, expectations play a crucial role and discretion leads to what is known as a stabilization bias. Clarida, Gali and Gertler (1999) then uses this result to propose the appointment of a conservative central banker (originally proposed by Rogoff (1985) to reduce the average inflation bias of discretionary policy) that puts more weight on inflation stabilization, that is, a smaller value of ‘\( \alpha \)’ to reduce this stabilization bias. The general idea here is that the outcomes of policy can be improved by assigning the central bank an objective function that differs from the optimally derived objective function (Walsh (1995)). A large body of literature has emerged that analyzes the impact of alternative central bank objective functions, termed as targeting regimes, on the outcome of policy. In what follows, I focus on two of such targeting regimes, namely, inflation targeting and
price-level targeting. The loss functions corresponding to these targeting regimes can be expressed respectively as:

\[ L_i = \frac{1}{2} E \left[ \sum_{i=0}^{\infty} \beta^i \left( \alpha_{IT} x_{t+i}^2 + \pi_{t+i}^2 \right) \right] \quad \text{(Inflation Targeting)} \quad (38) \]

and

\[ L_i = \frac{1}{2} E \left[ \sum_{i=0}^{\infty} \beta^i \left( \alpha_{PT} x_{t+i}^2 + p_{t+i}^2 \right) \right] \quad \text{(Price-level Targeting)} \quad (39) \]

where, ‘\( \alpha_{IT} \)’ is the weight on output stabilization relative to inflation stabilization and ‘\( \alpha_{PT} \)’ is the weight on output stabilization relative to price-level stabilization.

4.1)- Inflation Targeting

This is the most popular and widely studied targeting regime. Accordingly, I will treat the policy outcomes under inflation targeting as a benchmark and then compare them with the policy outcomes under price-level targeting. A similar approach was employed by Vestin (2000). The key questions I seek to answer are: First, what is the implication of the cost channel in comparing the outcomes of discretionary optimal monetary policy and commitment optimal policy? Second, whether optimal discretionary monetary policy under price-level targeting captures the outcome of optimal policy with commitment under an inflation targeting regime. The main difference between Vestin’s analysis and the analysis conducted in this paper is the presence of the cost channel. As will be evident below, with the presence of cost channel, the IS curve will not be irrelevant in deriving the optimal policy and the gains from commitment will be larger.

4.1.1)- Discretionary Case
In deriving the optimal discretionary monetary policy, it is assumed that the central bank uses nominal interest rate $i_t$ as its instrument variable. It chooses the time path of this instrument to influence the time paths of the target variables $x_t$ and $\pi_t$ in such a way that it maximizes equation (38) subject to the constraints on their behaviour implied by the system of equations, (33) – (36).

The solution to the constrained minimization exercise yields the following first order conditions:

$$\frac{\partial L}{\partial \pi_t} = \pi_t - \psi_t = 0$$  \hspace{1cm} (40)

$$\frac{\partial L}{\partial x_t} = \alpha_{it} x_t + \lambda \psi_t - \phi_t = 0$$  \hspace{1cm} (41)

$$\frac{\partial L}{\partial i_t} = \psi_t \phi_t - \sigma \phi_t = 0$$  \hspace{1cm} (42)

where, ‘$\psi_t$’ and ‘$\phi_t$’ are the Langrangian multipliers associated with the new Phillips curve and the new IS curve respectively.

The main difference between this model and the basic new Keynesian model is evident from equation (42). In the basic new Keynesian model ($\kappa = 0$), $\phi_t$ would be zero. This explains the reason why the new IS curve is usually ignored in deriving the optimal discretionary policy. However, with the cost channel present, that is, with $\kappa = 1$, $\phi_t$ is different from zero and the IS curve will be relevant.
Eliminating ‘ψ’ and ‘φ’ from the first order conditions, yields the following optimality condition:

\[ x_t = -\left( \frac{\sigma \lambda - \gamma \kappa}{\sigma \alpha_{t+1}} \right) \pi_t \]  \hspace{1cm} (43)

Equation (43) has the usual interpretation that the central bank should contract output by raising the interest rate whenever inflation is above target and vice versa (as long as \((\sigma \lambda > \gamma)\)). With \(\kappa = 0\) the equation is exactly the same as derived in Clarida, Gali and Gertler (1999) for optimal discretionary policy. However, the important difference emerges when the parameter ‘\(\kappa\)’, that depicts the effect of the cost channel of monetary policy, is set equal to one. It is straightforward to verify that the response of the central bank in trading-off fluctuations in the output for stabilization of inflation is less aggressive when \(\kappa = 1\) compared to when \(\kappa = 0\). Thus, the presence of cost channel makes inflation stabilization more costly. The reason for this is that as the central bank increases the interest rate to bring inflation down, output goes down that reduces inflation but at the same time it increases inflation directly. The net effect, however, would be that the inflation comes down because for all plausible parameter values the condition \((\sigma \lambda > \gamma)\) holds.

To obtain solution for the model that describe the equilibrium behaviour of \(\pi_t\) and \(x_t\), first eliminate \(i_t\) from equation (37) using equation (36) to get:

\[ \pi_t = (\beta + \gamma \kappa) E_t \pi_{t+1} + (\gamma \kappa / \sigma) E_t x_{t+1} + \left( (\sigma \lambda - \gamma \kappa) / \sigma \right) x_t + (\gamma \kappa / \sigma) u_t + v_t \]  \hspace{1cm} (44)
then combine this equation with the optimality condition (43) to find the reduced form expressions for $x_i$ and $\pi_i$ using the method of undetermined coefficients. The only relevant state variables are $u_t$ and $\nu_t$. Thus, the trial solution takes the form:

$$\pi_i = b_{IT}^d u_i + e_{IT}^d v_i$$

$$x_i = e_{IT}^d u_i + f_{IT}^d v_i$$

(45)

(46)

The solution expressions for $b_{IT}^d$, $c_{IT}^d$, $e_{IT}^d$ and $f_{IT}^d$ are given as follows:

$$b_{IT}^d = \frac{(\gamma \kappa / \sigma)}{q_1}$$

(47)

$$c_{IT}^d = \frac{1}{q_2}$$

(48)

$$e_{IT}^d = -\Omega_{IT} b_{IT}^d$$

(49)

$$f_{IT}^d = -\Omega_{IT} c_{IT}^d$$

(50)

where,

$$\Omega_{IT} = \left(\frac{\sigma \lambda - \gamma \kappa}{\sigma \alpha_{IT}}\right)$$

$$q_1 = 1 - \rho_s (\beta + \gamma \kappa) + \lambda \Omega_{IT} - (\gamma \kappa / \sigma)(1 - \rho_s) \Omega_{IT}$$

and

$$q_2 = 1 - \rho_s (\beta + \gamma \kappa) + \lambda \Omega_{IT} - (\gamma \kappa / \sigma)(1 - \rho_s) \Omega_{IT}$$

Note that, in the absence of cost channel ($\kappa = 0$), both $b_{IT}^d$ and $e_{IT}^d$ are equal to zero. This is a standard result in a basic new Keynesian model meaning that demand shock $u_t$ has no effect on output-gap and inflation and thus the central bank faces no trade-off in stabilizing output-gap and inflation. However, when the cost channel is operating ($\kappa = 1$), the demand shock $u_t$ will lead to a trade-off between output-gap fluctuation and inflation.
fluctuation. Consider a positive realization of $u_t$. As a result, both output-gap $x_t$ and inflation $\pi_t$ increases. The central bank responds by increasing the interest rate. This lowers the output-gap and thus inflation but also increases inflation directly. In order to stabilize both output-gap and inflation the central must trade-off some fluctuation in output-gap for a smaller fluctuation in inflation.

In order to draw comparisons across alternative monetary policy regimes, it would prove convenient to evaluate the performance of a policy by calculating the unconditional expected value of the loss function expressed in terms of variances of inflation and output-gap. Thus, the unconditional expected value of the loss function given by equation (38) can be approximately written as:

$$E(L_t^d) = \text{var}(\pi_t) + \alpha_{IT} \text{var}(x_t)$$

(51)

Using expressions (47) – (50), I can calculate the variance expressions for inflation and output-gap as follows:

$$\text{var}(\pi_t) = b_{IT}^{d^2} \text{var}(u_t) + c_{IT}^{d^2} \text{var}(v_t)$$

(52)

$$\text{var}(x_t) = e_{IT}^{d^2} \text{var}(u_t) + f_{IT}^{d^2} \text{var}(v_t)$$

(53)

where the variances for the demand and supply shocks are given as:

$$\text{var}(u_t) = \frac{1}{1 - \rho_u} \text{var}(\epsilon_t^u)$$

(54)

$$\text{var}(v_t) = \frac{1}{1 - \rho_v} \text{var}(\epsilon_t^v)$$

(55)

In order to learn more about the outcome of policy and its sensitivity to various parameter values, I use some specific parameter values to evaluate the discretionary case. The same
parameter values will then be used to evaluate the commitment case under inflation targeting and the discretionary case under price-level targeting. Calibrating the model in this fashion makes it easy to compare the properties of the model with and without the cost channel and highlight the significance of the cost channel of monetary policy. The following baseline parameter values are used:

\[ \beta = 0.99, \lambda = 0.3, \gamma = 0.1, \sigma = 0.67, \rho_u = 0.3, \rho_v = 0.3, \operatorname{var}(\epsilon^u) = \operatorname{var}(\epsilon^v) = 0.000225 \]

and

\[ \alpha = 0.01, 0.25 \text{ or } 1.0 \] and \( \kappa \) is set to zero if the cost channel is closed and 1 if the cost channel is operating. The results are reported in table 1 and 2 for both demand and cost-push shocks respectively.

### Table 1: Demand shock\(^{13}\)

<table>
<thead>
<tr>
<th></th>
<th>Cost channel closed</th>
<th>Cost channel operating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \kappa = 0 )</td>
<td>( \kappa = 1 )</td>
</tr>
<tr>
<td>( \alpha_{IT} = .01 )</td>
<td>.01</td>
<td>.01</td>
</tr>
<tr>
<td>( \alpha_{IT} = .25 )</td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td>( \alpha_{IT} = 1.0 )</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( \operatorname{var}(\pi_t) )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \operatorname{var}(x_t) )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \operatorname{Loss}(I^d_t) )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

It is obvious from the table that a demand shock implies an inflation-output-gap stabilization tradeoff in the presence of the cost channel. Thus, the central bank incurs positive losses in the presence of cost channel and these losses increases as the central

---

\(^{13}\) Reported values are multiplied by \(10^5\).
bank puts more weight on stabilizing output-gap. This has a simple intuitive explanation. For example, in case of a positive demand shock that increases the output-gap and thus inflation, the central bank responds by increasing the nominal interest rate. This lowers the output-gap and indirectly inflation. However, in the presence of cost channel this policy response also directly increases inflation. If the central bank puts a lot of weight on stabilizing output-gap then she would (relatively) ignore this extra volatility in inflation thus incurring some extra losses. Figure 1 captures this monetary policy trade-off between stabilizing inflation and output-gap. Note that this efficiency policy frontier is only possible in case of demand shocks when the cost channel is operating. Thus, the claim made by Clarida, Gali and Gertler (1999) that the central bank will face no trade-off in case of demand shocks is refuted by introducing the cost channel of monetary policy in a new Keynesian model.

\[
\text{Figure 1: Efficient Policy Frontier}
\]
The importance of the cost channel of monetary policy is further highlighted in the presence of cost-push shocks --- the losses incurred by the central bank increases for all values of alpha. With the exception of very small values of ‘$\alpha_{IT}$’ (approximately less than 0.05, as is evident from figure 3 below), the cost channel lowers the volatility in output-gap at the cost of increased volatility in inflation. For example, consider a positive cost-push shock that increases inflation. A typical response by the central bank is to increase the nominal interest rate. This policy response creates a negative output-gap and thus helps in lowering inflation. However, with the cost channel present, an increase in the interest rate pushes up the inflation even further. Thus, the central bank would be less aggressive in increasing the interest rate. This would lead to a smaller negative output-gap at the cost of higher inflation. Figure 2 and 3 compare the volatility in inflation and output-gap with and without the cost channel. The important point is that the cost of discretion is higher in the presence of the cost channel of monetary policy and this strengthens the case for optimal commitment policy even more.
Figure 4 and 5 compares the efficient policy frontier with and without the cost channel. It is evident that the policy frontier with the cost channel (figure 5) is higher than the policy frontier without the cost channel (figure 4) for all values of alpha. Thus, it is more ‘costly’ to stabilize inflation and output-gap.
The Implicit Interest rate rule

The interest rate rule that implements the discretionary monetary policy can be derived by using the optimality condition (equation 43), the solution expressions for $\pi_t$ and $x_t$ as given by equations (45) and (46) and the IS relationship given by equation (33). It is given as follows:
\[ i_t = \left( 1 + \frac{\Omega_{tt} (1 - \rho_v)}{\sigma \rho_v} \right) E_t \pi_t + \left( \frac{1}{\sigma} + \frac{b^d_{tt} \Omega_{tt} (1 - \rho_u)}{\sigma} + \frac{b^d_{tt} \rho_u \Omega_{tt} (1 - \rho_v)}{\sigma \rho_v} \right) u_t \]  

(56)

With no cost channel, equation (56) reduces to exactly the same expression as derived in Clarida, Gali and Gertler (1999) since \( b^d_{tt} = 0 \) and \( \Omega_{tt} \) reduces to \( \lambda / \alpha \) implying that the demand shocks is perfectly offset and the central bank faces a tradeoff between stabilizing inflation and output-gap only in the presence of a cost-push shock. Moreover, in response to a one percent change in the expected inflation rate, the central bank should change the nominal interest rate by more than one percent (as evident from the coefficient in front of the expected inflation term) so that the real interest rate is affected in the ‘right’ direction. This principal ensures determinacy and is called the Taylor principal.

In the presence of the cost channel, however, the demand shock is not completely offset as \( b^d_{tt} \neq 0 \). Also, the response of the central bank to expected movements in the inflation rate is less aggressive. That is, the central bank changes the nominal interest rate by a smaller amount (still greater than one) in response to expected inflation. The reason for this, as explained above, is that now a change in the interest rate directly affects inflation in addition to its indirect effect through output-gap.

An alternative method of deriving the interest rate rule is to solve the IS relationship (equation (33)), the Phillips curve (equation (34)) and the central bank’s optimality condition (equation (43)) simultaneously for \( i_t, \pi_t \) and \( x_t \). Although in the discretionary case, there is no difference in the two methods, however, in case of commitment it makes a difference as will be demonstrated shortly. Using this alternative approach, the solution
expressions for inflation and output-gap remain the same as described by equation (45) and (46) and the expression for the implicit interest rate rule is determined as follows:

$$i_t = \left( \frac{1}{\sigma} + b^d_{\pi} \Omega_{\pi} (1 - \rho_u) + b^d_{\pi} \rho_u \right) u_t + \left( c^d_{\pi} \Omega_{\pi} (1 - \rho_u) + c^d_{\pi} \rho_u \right) \psi_t$$

(57)

### 4.1.2)- Commitment Case

This section demonstrates that there are gains from commitment even in the absence of inflation bias and that these gains are larger with the cost channel present. Under the commitment policy, central bank seeks to minimize the loss function given in (38) by choosing the current and future values of inflation, output-gap, and the nominal interest rate subject to the constraints, equation (33) – (36). The first order conditions for this minimization exercise are:

$$\frac{\partial L_t}{\partial \pi_t} = \pi_t - \psi_t = 0 \quad \text{for } t = 0$$

(58)

$$\frac{\partial L_t}{\partial \pi_t} = \beta \pi_t + \beta \psi_{t-1} - \beta \psi_t + \sigma \phi_{t-1} = 0 \quad \text{for } t \geq 1$$

(59)

$$\frac{\partial L_t}{\partial x_t} = \alpha_{\pi} x_t + \lambda \psi_t - \phi_t = 0 \quad \text{for } t = 0$$

(60)

$$\frac{\partial L_t}{\partial x_t} = \alpha_{\pi} \beta x_t + \beta \lambda \psi_t + \phi_{t-1} - \beta \phi_t = 0 \quad \text{for } t \geq 1$$

(61)

$$\frac{\partial L_t}{\partial i_t} = \gamma \kappa \psi_t - \sigma \phi_t = 0 \quad \text{for } t = 0$$

(62)

$$\frac{\partial L_t}{\partial i_t} = \beta \gamma \kappa \psi_t - \beta \sigma \phi_t = 0 \quad \text{for } t \geq 1$$

(63)
Eliminating the Langrangian multipliers, \( \psi \) and \( \phi \), I can simplify the first order conditions to get:

\[
x_t = -(\frac{\sigma \lambda - \gamma k}{\sigma \alpha_{IT}})\pi_t \quad \text{for} \quad t = 0 \tag{64}
\]

\[
x_t = x_{t-1} - (\frac{\sigma \lambda - \gamma k}{\sigma \alpha_{IT}})\pi_t \quad \text{for} \quad t \geq 1 \tag{65}
\]

Thus, as reported by Clarida, Gali and Gertler (1999), Woodford (2000) and McCallum and Nelson (2000), the optimal commitment policy entails inertial behaviour; rather than adjusting the level of output-gap in response to fluctuations in inflation, the commitment policy requires the adjustment in the change in the output gap to changes in inflation.

However, in the initial period, \( t = 0 \), the central bank behaves as if it were operating in a discretionary fashion. Thus, the commitment solution is not time-consistent. To get around this problem Woodford (1999) has suggested that the central bank should commit to implement in each period the policy that it would have been optimal to commit to if the same problem had been considered at a date far in the past. This procedure avoids treating the current period \( t = 0 \) as the initial one by setting inflation in that period as if it were one of many future periods when policy was considered in the distant past. Woodford (1999) labels this approach the timeless perspective of monetary policy. \(^{14}\)

In the context of the present model, the timeless perspective policy amount to implementing equation (65) for all time periods including the initial period. The basic idea behind this timeless perspective pre-commitment policy is that the policymaker commits to a policy

\(^{14}\) It is important to note that a number of authors such as Dennis (2001), Blake (2001) and Jensen and McCallum (2002) have questioned the timelessness of Woodford’s approach and have demonstrated that timeless perspective policy is not the time-invariant rule and does not in general minimizes the loss function. This issue, though important, is beyond the scope of the present paper.
that disregards the conditions that happen to prevail at the time in which the policy begins.

The gains from commitment with the cost channel present are easily understood by looking at equation (44):

\[
\pi_t = (\beta + \gamma \kappa) E_t \pi_{t+1} + (\gamma \kappa / \sigma) E_t x_{t+1} + ((\sigma \lambda - \gamma \kappa) / \sigma) \pi_t + (\gamma \kappa / \sigma) \mu_t + v_t
\]  

(44)

With the cost channel operating, the future expected output gap also affects the current inflation in addition to current output-gap and future expected inflation. The reason the expected output-gap \( E_t x_{t+1} \) directly affects current inflation \( \pi_t \) is because a lower \( E_t x_{t+1} \) reduces the nominal interest rate associated with any given current output-gap. And since the nominal interest rate directly effects \( \pi_t \) because of the cost channel, current inflation goes down. Also, \( E_t x_{t+1} \) will affect \( E_t \pi_{t+1} \) that in turn affects \( \pi_t \) in addition to the usual effect. Now consider a positive demand shock that pushes up the current output-gap and inflation. The central bank increases the nominal interest rate. As a result current output-gap goes down also reducing inflation. With the cost channel, this increased interest rate also increases inflation directly. Up until now the analysis is the same as in the discretionary case. However, with commitment policy, the central bank does not take the expectations as given, so they also affect current behaviour of inflation and output-gap. More precisely, the expected output-gap also decreases when the central bank increases the interest rate as long as the shock is serially correlated. This fall in the expected output-gap will lower current inflation as explained above (using equation (44)). Moreover, a fall in the expected output-gap will also lower future expected inflation that also directly affects inflation. With the cost channel operating the effect of future
expected inflation on current inflation is bigger. Thus, inflation goes down by a bigger amount --- due to a fall in the current output-gap and the expectations of a lower future expected output-gap and inflation --- than it goes up due to an increase in the interest rate. The direct effect of expected future output gap and its indirect effects via future expected inflation on current inflation are only operational in the presence of the cost channel and are precisely the reason why the gains from commitment increase.

In order to obtain the solution with optimal commitment policy (which is the timeless perspective policy of Woodford), I combine equation (44) (the inflation adjustment equation after substituting for $i$, using equation (33)) with equation (65). The trial solution used for this purpose is given as:

$$\pi_t = a_{it}^c x_{t-1} + b_{it}^c u_t + c_{it}^c v_t$$  \hspace{1cm} (66)

$$x_t = d_{it}^c x_{t-1} + e_{it}^c u_t + f_{it}^c v_t$$  \hspace{1cm} (67)

The solution expressions for $a_{it}^c, b_{it}^c, c_{it}^c, d_{it}^c, e_{it}^c$, and $f_{it}^c$ are given as:

$$a_{it}^c = (\beta + \gamma \kappa) a_{it}^c d_{it}^c + (\gamma \kappa / \sigma) d_{it}^c + ((\sigma \lambda - \gamma \kappa) / \sigma) d_{it}^c$$  \hspace{1cm} (68)

$$b_{it}^c = (\beta + \gamma \kappa) (a_{it}^c e_{it}^c + \rho \sigma b_{it}^c) + (\gamma \kappa / \sigma) (d_{it}^c e_{it}^c + \rho \sigma e_{it}^c) + ((\sigma \lambda - \gamma \kappa) / \sigma) e_{it}^c + (\gamma \kappa / \sigma)$$  \hspace{1cm} (69)

$$c_{it}^c = (\beta + \gamma \kappa) (a_{it}^c f_{it}^c + \rho \sigma c_{it}^c) + (\gamma \kappa / \sigma) (d_{it}^c f_{it}^c + \rho \sigma f_{it}^c) + ((\sigma \lambda - \gamma \kappa) / \sigma) f_{it}^c + 1$$  \hspace{1cm} (70)

$$d_{it}^c = 1 - \Omega_{it} a_{it}^c$$  \hspace{1cm} (71)

$$e_{it}^c = -\Omega_{it} b_{it}^c$$  \hspace{1cm} (72)

$$f_{it}^c = -\Omega_{it} c_{it}^c$$  \hspace{1cm} (73)
It is obvious from the above expressions that their solution would involve multiple values for $a_{IT}$ and thus for all other undetermined coefficients. The solution for $a_{IT}$ can be obtained by solving the following quadratic equation:

$$q_3a_{IT}^2 + q_4a_{IT} - \lambda = 0$$

(74)

where

$$q_3 = (\beta + \gamma \kappa)\Omega_{IT} - (\gamma \kappa / \sigma)\Omega_{IT}^2$$

and

$$q_4 = 1 - (\beta + \gamma \kappa) + \lambda \Omega_{IT} - (\gamma \kappa / \sigma)\Omega_{IT}$$

The solution for equation (74) that satisfies $0 < a_{IT} < 1$ (and thus $0 < d_{IT} < 1$) is given as:

$$a_{IT} = \frac{-q_4 + \sqrt{q_4^2 + 4\lambda q_3}}{2q_3}$$

(75)

Accordingly, $b_{IT}$ and $c_{IT}$ are determined as follows. $d_{IT}^c, e_{IT}^c$ and $f_{IT}^c$ can then be determined from equations (71) – (73):

$$b_{IT} = \frac{(\gamma \kappa / \sigma)}{q_5}$$

(76)

$$c_{IT} = \frac{1}{q_6}$$

(77)

where,

$$q_5 = 1 + (\beta + \gamma \kappa)(a_{IT}^c + \Omega_{IT} - \rho_u) + \Omega_{IT} \left(\left(\sigma A - \gamma \kappa\right)/\sigma\right) + (\gamma \kappa / \sigma)(d_{IT}^c + \rho_u)$$

$$q_6 = 1 + (\beta + \gamma \kappa)(a_{IT}^c + \Omega_{IT} - \rho_v) + \Omega_{IT} \left(\left(\sigma A - \gamma \kappa\right)/\sigma\right) + (\gamma \kappa / \sigma)(d_{IT}^c + \rho_v)$$

The variance of inflation and output-gap can now be calculated using equation (66) and (67):
\[ \text{var}(\pi_i) = \left( \frac{a^{c^2}_{\pi}e^{c^2}_{\pi}(1 + \rho_u d^{c}_{\pi})}{(1 - d^{c^2}_{\pi})(1 - \rho_u d^{c}_{\pi})} + b_{\pi}^2 + \frac{2a_{\pi}^{c^2}b_{\pi}^{c^2}e^{c^2}_{\pi}\rho_u}{1 - \rho_u d^{c}_{\pi}} \right) \text{var}(u_i) \]

\[ + \left( \frac{a^{c^2}_{\pi} f^{c^2}_{\pi}(1 + \rho_u d^{c}_{\pi})}{(1 - d^{c^2}_{\pi})(1 - \rho_u d^{c}_{\pi})} + c_{\pi}^2 + \frac{2a_{\pi}^{c^2} f^{c^2}_{\pi}\rho_v}{1 - \rho_v d^{c}_{\pi}} \right) \text{var}(v_i) \]  

(78)

\[ \text{var}(x_i) = \frac{1}{(1 - d^{c^2}_{\pi})} \left( \frac{e^{c^2}_{\pi}(1 + d^{c^2}_{\pi}\rho_u)}{1 - \rho_u d^{c}_{\pi}} \text{var}(u_i) + \frac{f^{c^2}_{\pi}(1 + \rho_u d^{c}_{\pi})}{1 - \rho_u d^{c}_{\pi}} \text{var}(v_i) \right) \]  

(79)

Thus, the unconditional expected value of the loss function in the commitment case, given by equation (38), can be calculated as:

\[ E(L_i^c)^{\pi} = \text{var}(\pi_i) + \alpha_{\pi} \text{var}(x_i) \]  

(80)

Using the parameter values mentioned above, the calibrated results of the model for demand and cost-push shock are reported respectively in table 3 and table 4:

**Table 3: Demand shock**

<table>
<thead>
<tr>
<th></th>
<th>Cost channel closed</th>
<th>Cost channel operating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(( \kappa = 0 ))</td>
<td>(( \kappa = 1 ))</td>
</tr>
<tr>
<td>( \alpha_{\pi} )</td>
<td>.01</td>
<td>.01</td>
</tr>
<tr>
<td>( \alpha_{\pi} )</td>
<td>.25</td>
<td>.01</td>
</tr>
<tr>
<td>( \alpha_{\pi} )</td>
<td>1.0</td>
<td>.25</td>
</tr>
<tr>
<td>( \alpha_{\pi} )</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>( \text{var}(\pi_i) )</td>
<td>0</td>
<td>0.036</td>
</tr>
<tr>
<td>( \text{var}(x_i) )</td>
<td>0</td>
<td>7.90</td>
</tr>
<tr>
<td>Loss( (L_i^c) )</td>
<td>0</td>
<td>0.115</td>
</tr>
</tbody>
</table>

Although the cost channel leads to an inflation-output-gap volatility tradeoff, but quantitatively the effects are small. To appreciate the importance of cost channel and gains from commitment compare the losses reported in table 1 and table 3. For example,
when $\alpha_{IT} = 0.25$, the expected loss decreases by almost 50% due to commitment. Thus, it confirms the result that there are gains from commitment even in the absence of inflation bias and that these gains are larger in the presence of cost channel.

**Table 4: Cost-push shock**

<table>
<thead>
<tr>
<th></th>
<th>Cost channel closed</th>
<th>Cost channel operating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($\kappa = 0$)</td>
<td>($\kappa = 1$)</td>
</tr>
<tr>
<td>$\alpha_{IT} = .01$</td>
<td>$\alpha_{IT} = .25$</td>
<td>$\alpha_{IT} = 1.0$</td>
</tr>
<tr>
<td>$\alpha_{IT} = .01$</td>
<td>$\alpha_{IT} = .25$</td>
<td>$\alpha_{IT} = 1.0$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>var($\pi_t$)</th>
<th>var($x_t$)</th>
<th>Loss($L^*_t$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\kappa = 0)$</td>
<td>0.377</td>
<td>274.235</td>
<td>3.119</td>
</tr>
<tr>
<td>$(\kappa = 1)$</td>
<td>15.367</td>
<td>41.491</td>
<td>25.739</td>
</tr>
<tr>
<td></td>
<td>30.375</td>
<td>9.361</td>
<td>39.736</td>
</tr>
<tr>
<td></td>
<td>1.607</td>
<td>354.606</td>
<td>5.153</td>
</tr>
<tr>
<td></td>
<td>22.537</td>
<td>20.339</td>
<td>27.621</td>
</tr>
<tr>
<td></td>
<td>36.082</td>
<td>3.341</td>
<td>39.423</td>
</tr>
</tbody>
</table>

Again, the impact of the cost channel on the losses of the central bank is quite small and insignificant in the commitment case. However, this does not undermine the result that the presence of cost channel strengthens the case for commitment. Comparing the losses reported in table 2 and 4 reveals that the losses are lower with commitment. As in the case of demand shock, when $\alpha_{IT} = 0.25$, the expected loss decreases by almost 50% due to commitment in case of cost-push shocks as well.
Figure 6 – Inflation volatility

Figure 7 – Output-gap volatility

Figure 8: Efficient Policy Frontier **without** cost channel
The Implicit Interest rate rule

Following the method of Clarida, Gali and Gertler (1999), I can derive the interest rate rule that implements the optimal pre-commitment monetary policy by writing the central bank’s optimality condition (equation (65)) one period forward, taking expectations and substituting the result in the IS relationship (equation (33)).

\[
 i_t = \left(1 - \frac{\Omega \pi_t}{\sigma}\right) E_t \pi_{t+1} + \frac{1}{\sigma} u_t,
\]  

(81)

Two points are worth noting about equation (81). First, the coefficient associated with expected inflation is less than one. Accordingly, an increase in expected inflation leads to a small (less than one) increase in the nominal interest rate implying that the real interest rate move in the ‘wrong’ direction, that is, it may actually decrease. Clarida, Gali and Gertler (1999) argue that a rule of this type violate the Taylor principle and may lead to indeterminacies of output and inflation. Second, the rule in equation (81) implies that the demand shocks will be completely stabilized whether the cost channel is present or not.
This implication is inconsistent with the result derived and discussed above that the demand shock cannot be completely stabilized in the presence of cost channel. The reason for this ambiguity is the method of deriving equation (81). To be specific, the following version of the optimality condition (equation (65)) is used to get equation (81):

$$E_{t}x_{t+1} = x_{t} - \left( \frac{\sigma^{*} - \gamma \kappa}{\sigma \alpha_{tt}} \right) E_{t} \pi_{t+1}$$  \hspace{1cm} (65a)

But, it is important to note that while equation (65) implies equation (65a), the opposite is not true. Equation (65) may not hold even if equation (65a) does. Thus, the interest rate rule implied by equation (81) does not implement optimality condition (65). For this reason and to confirm the possibility of indeterminacy, I use an alternative method (described in the discretionary case) to derive the interest rate rule that implements the optimality condition given by equation (65).

The solution method involves solving the IS relationship (equation (33)), the Phillips curve (equation (34)) and the optimality condition --- equation (65) simultaneously for $i_{t}, \pi_{t}$ and $x_{t}$ using the following trial solutions:

$$\pi_{t} = a_{tt}^{c}x_{t-1} + b_{tt}^{c}u_{t} + c_{tt}^{c}v_{t}$$  \hspace{1cm} (66)

$$x_{t} = d_{tt}^{c}x_{t-1} + e_{tt}^{c}u_{t} + f_{tt}^{c}v_{t}$$  \hspace{1cm} (67)

$$i_{t} = g_{tt}^{c}x_{t-1} + h_{tt}^{c}u_{t} + f_{tt}^{c}v_{t}$$  \hspace{1cm} (82)

Not surprisingly, the expressions for $a_{tt}^{c}, b_{tt}^{c}, c_{tt}^{c}, d_{tt}^{c}, e_{tt}^{c}$ and $f_{tt}^{c}$ turns out to be the same as defined by equations (75), (76), (77), (71), (72) and (73) respectively. The implied interest rate rule is given as follows:
\[ i_t = \left(1 - a_{\pi}^c \Omega_{\pi}\right) \left(\frac{\alpha_{\pi}^c (\sigma - \Omega_{\pi})}{\sigma}\right)x_{t-1} + \frac{1}{\sigma} + \frac{\gamma \kappa}{\sigma} \left(\rho_{u} - a_{\pi}^c \Omega_{\pi}\right) \left(\frac{\sigma - \Omega_{\pi}}{\sigma q_6}\right) \right) u_t \\
+ \left(\rho_{v} - a_{\pi}^c \Omega_{\pi}\right) \left(\frac{1 + \Omega_{\pi}}{q_6}\right) v_t \]  

Equation (83) offers lot of insight in the implementation of optimal pre-commitment monetary policy. The first important point to note is that, unlike equation (81), it captures the inertial behaviour of the central bank implied by the optimality condition (65). The second point is that the coefficient in front of the lagged output-gap term must be positive to avoid self-fulfilling fluctuations in output and inflation; current interest rate must increase if in the previous period output-gap goes up. This condition is satisfied if \((\sigma - \Omega_{\pi}) > 0\). It is easy to verify that this condition may be violated for small values of \(\alpha_{\pi}\). Similarly, \(\left(\rho_{v} - a_{\pi}^c \Omega_{\pi}\right)\) must be positive if the interest rate were to increase in response to a positive cost-push shock. Lastly, equation (83) confirms the previous result that the central bank cannot completely stabilize both output-gap and inflation in response to demand shocks because the coefficient of \(u_t\) is different from \(1/\sigma\) in the presence of cost channel.

4.2)- Price-level Targeting

Assuming that in practice no central bank can commit to a policy rule for all time periods and acknowledging at the same time that there are gains from commitment (as demonstrated in the above section), it is a reasonable question to ask: can we delegate the central bank a targeting regime that would replicate the commitment solution while operating in a discretionary fashion? Vestin (2000) answers this question in an
affirmative, provided the cost-push shock is serially uncorrelated. In this section I reconsider the question in a more general model in which demand shocks are also relevant due to the presence of cost channel of monetary policy.

The central bank seeks to minimize the following loss function subject to the constraints implied by equation (33)- (36):

\[
L_t = \frac{1}{2} E_t \left[ \sum_{i=0}^{\infty} \beta^i \left( \alpha_{pt} x_{t+i}^2 + p_{t+i}^2 \right) \right]
\]  

(39)

Since the loss function is in terms of the price-level rather than the inflation rate, it would be useful to express the model in terms of price-level as well. Thus, the new IS curve and the new inflation adjustment equation are written as follows:

\[
x_t = E_t x_{t+1} - \sigma \left( i_t - (E_t p_{t+1} - p_t) \right) + u_t
\]  

(84)

\[
p_t - p_{t-1} = \beta \left( E_t p_{t+1} - p_t \right) + \lambda x_t + \theta \kappa i_t + v_t
\]  

(85)

The only new first order condition will be with respect to the price-level; the other two (w.r.t \( x_t \) and \( i_t \)) will be the same as in the inflation targeting case with discretion:

\[
\frac{\partial L_t}{\partial p_t} = p_t - (1 + \beta) \psi_t - \sigma \phi_t + E_t \psi_{t+1} = 0
\]  

(86)

\[
\frac{\partial L}{\partial x_t} = \alpha_{pr} x_t + \lambda \psi_t - \phi_t = 0
\]  

(87)

\[
\frac{\partial L}{\partial i_t} = \psi_t \gamma \kappa - \sigma \phi_t = 0
\]  

(88)

Combining these equations will give the following optimality condition describing the behaviour of the central bank:
\[ x_t = \left( \frac{1}{1 + \beta + \gamma \kappa} \right) E_{x_{t+1}} - \left( \frac{\sigma \lambda - \gamma \kappa}{\sigma \alpha_{pt} (1 + \beta + \gamma \kappa)} \right) p_t \]  

An interesting point to note from equation (89) is that it depicts the inertial behaviour of the central bank, which is a feature of the commitment solution implying the possibility that if a relative weight of \( \alpha_{pt} \) is assigned to the central bank instead of \( \alpha_{it} \), the commitment solution can be replicated. However, that would be a premature conclusion.

To be concrete, I solve the model using the method of undetermined coefficients with the following trial solutions:

\[ p_t = a_{pt} p_{t-1} + b_{pt} u_t + c_{pt} v_t \]  

\[ x_t = d_{pt} p_{t-1} + e_{pt} u_t + f_{pt} v_t \]  

Combining equation (85) (after eliminating \( i_t \) using equation (84)) and equation (89) with the trial solutions yields the following identifying restrictions:

\[ a_{pt} = \frac{1}{1 + \beta + \gamma \kappa} \left( 1 + (\beta + \gamma \kappa) a_{pt}^2 + (\sigma \lambda - \gamma \kappa)/\sigma d_{pt} + (\gamma \kappa/\sigma) a_{pt} d_{pt} \right) \]  

\[ b_{pt} = \frac{1}{1 + \beta + \gamma \kappa} \left( (\beta + \gamma \kappa)(a_{pt} b_{pt} + \rho_a b_{pt}) + (\sigma \lambda - \gamma \kappa)/\sigma e_{pt} + (\gamma \kappa/\sigma)(b_{pt} d_{pt} + \rho_a e_{pt}) + \gamma \kappa/\sigma \right) \]  

\[ c_{pt} = \frac{1}{1 + \beta + \gamma \kappa} \left( (\beta + \gamma \kappa)(a_{pt} c_{pt} + \rho_a c_{pt}) + (\sigma \lambda - \gamma \kappa)/\sigma f_{pt} + (\gamma \kappa/\sigma)(c_{pt} d_{pt} + \rho_a f_{pt}) + 1 \right) \]  

\[ d_{pt} = \frac{-\Omega_{pt} a_{pt}}{(1 + \beta + \gamma \kappa)(1 - a_{pt})} \]  

\[ e_{pt} = \frac{b_{pt} (d_{pt} - \Omega_{pt})}{(1 + \beta + \gamma \kappa)(1 - \rho_a)} \]  

\[ f_{pt} = \frac{c_{pt} (d_{pt} - \Omega_{pt})}{(1 + \beta + \gamma \kappa)(1 - \rho_c)} \]  

where,
\[ \Omega_{pt} = \frac{(\sigma \lambda - \gamma \kappa)}{\sigma \alpha_{pt}} \]

It is difficult to derive an analytical expression for unique values of these coefficients. Thus, I resort to the calibrated version of the model and pick a solution that satisfies \( 0 < a_{pt} < 1 \).

The variance of the price-level and output-gap can now be calculated using equation (90) and (91):

\[
\begin{align*}
\var(x_t) &= \left( \frac{b_{pt}^2 d_{pt}^2 (1 + \rho_u a_{pt})}{(1 - a_{pt})(1 - \rho_u a_{pt})} + c_{pt}^2 + \frac{2b_{pt} d_{pt} e_{pt} \rho_u}{1 - \rho_u a_{pt}} \right) \var(u_t) \\
&\quad + \left( \frac{c_{pt}^2 d_{pt}^2 (1 + \rho_v a_{pt})}{(1 - a_{pt})(1 - \rho_v a_{pt})} + f_{pt}^2 + \frac{2c_{pt} d_{pt} f_{pt} \rho_v}{1 - \rho_v a_{pt}} \right) \var(v_t) \\
&= \left( \frac{2b_{pt}^2 (1 + \rho_u a_{pt})}{(1 + a_{pt})(1 - \rho_u a_{pt})} \right) \var(u_t) + \left( \frac{2c_{pt}^2 (1 + \rho_v a_{pt})}{(1 + a_{pt})(1 - \rho_v a_{pt})} \right) \var(v_t)
\end{align*}
\] (98)

\[
\var(p_t) = \frac{1}{(1 - a_{pt}^2)} \left( \frac{b_{pt}^2 (1 + a_{pt} \rho_u)}{1 - \rho_u a_{pt}} \right) \var(u_t) + \left( \frac{c_{pt}^2 (1 + \rho_v a_{pt})}{1 - \rho_v a_{pt}} \right) \var(v_t)
\] (99)

The implied variance of inflation under price-level targeting is given as follows:

\[
\var(\pi_t) = \left( \frac{2b_{pt}^2 (1 - \rho_u)}{(1 + a_{pt})(1 - \rho_u a_{pt})} \right) \var(u_t) + \left( \frac{2c_{pt}^2 (1 - \rho_v)}{(1 + a_{pt})(1 - \rho_v a_{pt})} \right) \var(v_t)
\] (100)

The unconditional expected value of the loss function with price-level targeting can be calculated as:

\[
E(L_t)^{pt} = \var(p_t) + \alpha_{pt} \var(x_t)
\] (101)

It would be incorrect to compare the absolute value of this expected loss under price-level targeting with expected loss under inflation targeting with pre-commitment (given by equation (80)) because the relative weights in the two expressions --- \( \alpha_{it} \) and \( \alpha_{pt} \) --- represent different benchmarks. \( \alpha_{it} \) is the relative weight assigned to the variability of
output-gap compared to the variability of inflation, while $\alpha_{pt}$ is the relative weight placed on the variability of output-gap compared to the variability of the price-level. As noted by Vestin (2000), failing to appreciate this difference may have created a bias in favour of the free-lunch result of Svensson (1999). However, using the expression for variance of inflation (equation (100)) and variance of the price-level (equation (99)), I can establish a link between $\alpha_{it}$ and $\alpha_{pt}$. Taking the ratio of the two variance expressions, I get:

$$\frac{\text{var}(\pi_t)}{\text{var}(p_t)} = \frac{(1 - a_{pt})\left[2b_{pt}^2(1 - \rho_u)(1 - a_{pt}\rho_v)\text{var}(u_t) + 2c_{pt}^2(1 - \rho_v)(1 - a_{pt}\rho_u)\text{var}(v_t)\right]}{b_{pt}^2(1 + a_{pt}\rho_u)(1 - a_{pt}\rho_v)\text{var}(u_t) + c_{pt}^2(1 + a_{pt}\rho_v)(1 - a_{pt}\rho_u)}$$

(102)

Using equation (102) I can calculate the following loss function:

$$E(L_t^{pt}) = \text{var}(\pi_t) + \alpha_{pt}^* \text{var}(x_t)$$

(103)

where

$$\alpha_{pt}^* = \frac{(1 - a_{pt})\left[2b_{pt}^2(1 - \rho_u)(1 - a_{pt}\rho_v)\text{var}(u_t) + 2c_{pt}^2(1 - \rho_v)(1 - a_{pt}\rho_u)\text{var}(v_t)\right]}{b_{pt}^2(1 + a_{pt}\rho_u)(1 - a_{pt}\rho_v)\text{var}(u_t) + c_{pt}^2(1 + a_{pt}\rho_v)(1 - a_{pt}\rho_u)} \cdot \alpha_{pt}$$

Now, the two expected losses, given by equation (80) and equation (103), can be compared.

The calibrated values for variance of price-level, inflation and output-gap and for the expected losses in case of demand and cost-push shocks are reported in table 5 and 6 respectively.
In addition to the result that demand shocks matter even with price-level targeting, an important result is that the price-level targeting is preferred over inflation targeting. For example, Comparing the expected losses in table 5 (the last row) with those reported in table 3 reveals that the losses decreases by almost 50% when the central bank targets price-level in a discretionary manner as compared to a pre-commitment inflation target. This confirms the previous results obtained by Vestin (2000) and Dittmar and Gavin (2000). However, it is worth noting that these authors derived this result for a cost-push shock only; in case of demand shocks central bank is indifferent in choosing between the two regimes. In this sense the result that price-level targeting regime is preferred over an inflation targeting regime in the presence of a demand shock can be considered a new result. Note that if the relative weight on inflation stability versus output-gap stability is not appropriately adjusted, the result could be opposite --- an inflation targeting regime...
would be preferred. (compare the second last row in table 5 with the expected losses in table 3).

Table 6: Cost-push shock

<table>
<thead>
<tr>
<th></th>
<th>Cost channel closed</th>
<th>Cost channel operating</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \kappa = 0 )</td>
<td>( \kappa = 1 )</td>
</tr>
<tr>
<td>( \alpha_{IT} = .01 )</td>
<td>( \alpha_{IT} = .25 )</td>
<td>( \alpha_{IT} = 1.0 )</td>
</tr>
<tr>
<td></td>
<td>var(( p_t ))</td>
<td>0.798</td>
</tr>
<tr>
<td></td>
<td>var(( x_t ))</td>
<td>210.532</td>
</tr>
<tr>
<td></td>
<td>var(( \pi_t ))</td>
<td>0.910</td>
</tr>
<tr>
<td>( Loss(L_t)^{PT} )</td>
<td>2.903</td>
<td>31.962</td>
</tr>
<tr>
<td>( Loss(L_t)^{PT} )</td>
<td>3.111</td>
<td>14.918</td>
</tr>
</tbody>
</table>

As obvious from the comparison between the expected values of losses in table 6 and table 4, price-level targeting is preferred over inflation targeting confirming the results reported in previous studies. However, it is important to recognize that this result would not hold if the relative weight in the loss function is not appropriately adjusted. Put differently, most of the earlier analyses (for example, Dittmar and Gavin (2000) and Svensson (1999)) while comparing the benefits of inflation targeting and price-level targeting, ignore this point. Had I followed the same strategy I would have ended up concluding that inflation targeting is preferable to price-level targeting when the cost channel is introduced.
5- Concluding Remarks

Relying on the ample empirical evidence and a number of previous theoretical flexible-price models the paper developed a model, in the tradition of the “new Keynesian” literature on monetary policy, to introduce the bank lending channel or the cost channel of monetary policy. The “new Keynesian” model has emerged as the standard framework, because this compact structure combines three very desirable features. First, it is firmly grounded in inter-temporal optimization, so the desire on the part of “new Classicals” for well-articulated micro-foundations is respected. With each equation being structural, the Lucas critique can be respected as the model is applied to policy questions. Second, a degree of nominal rigidity is allowed for, so the mechanism that Keynesians regard as essential for generating short-run real effects from demand shocks is an integral part of the structure. Third, the model is conveniently analyzed at the same level of aggregation as was common with earlier generations of policy-oriented discussions. One of the contributions of the present paper is that it analyzes both the traditional and the cost channel of monetary policy in one unified framework, therefore, embracing the strand of literature that studies only the cost channel in the folds of the new Keynesian framework.

The model is applied to two basic questions commonly analyzed in the context of optimal monetary policy: performance of discretionary policy versus pre-commitment policy, and relative benefits of inflation targeting versus price-level targeting. A number of important results emerged.
First, with the introduction of cost channel the demand shock leads to a tradeoff between stabilizing inflation and output-gap in addition to cost-push shocks. This result is in stark contrast compared to a widely accepted result reported by Clarida, Gali and Gertler (1999) that only cost-push shocks generate meaningful monetary policy problems.

Second, the gains from pre-commitment exist even in the absence of an inflation bias and these gains are larger due to the presence of the cost channel of monetary policy thus strengthening the case for commitment.

Third, the paper proposes an alternative method of calculating the implicit interest rate rule that implements the optimal monetary policy. It is shown that this alternative method is the correct method in the commitment case for the sake of internal consistency of results. For example, the interest rate rule in the commitment case, when derived using the existing approach, suggests that the demand shock is completely stabilized even in the presence of the cost channel. However, the calibrated results demonstrate the opposite thus implying an internal inconsistency of results. On the other hand when the alternative method proposed in the paper is used, this problem is resolved.

Fourth, the paper confirms the earlier results that price-level targeting with discretion is preferable over inflation targeting with commitment even in the presence of the cost channel. However, unlike the previous work, the paper is careful in comparing the expected values of the losses incurred in the two regimes by adjusting the relative weight
on output-gap stabilization as compared to inflation stabilization is appropriately adjusted. Using the commonly employed approach gives the opposite result.

In the end, I agree with McCallum when he concluded while comparing the performance of inflation targeting and nominal income targeting, “This demonstration does not establish that nominal income targeting is preferable to inflation targeting or to other rules for monetary policy. To reach such a conclusion would require an extensive combination of theoretical and empirical analyses, conducted in a manner that gives due emphasis to the principle of robustness to model specification, plus attention to concerns involving policy transparency and communication with the public”. The point of this paper was not to attempt any such ambitious undertaking. However, the results can be considered as a small step in that direction.

References


