Candy may not be a brain food, but first and second graders who worked in a simulated candy factory learned to add and subtract fluently and with understanding—without needing their teacher to tell them how to solve problems. This two-year project had several goals. First, like most teachers, I wanted my students to become fluent in addition and subtraction. *Principles and Standards for School Mathematics* (NCTM 2000) states that K–2 students should “develop efficient and accurate strategies . . . with a particular focus on two-digit numbers” (p. 79).

A second goal was to have the students develop an understanding of computation that was strong enough to solve more than routine problems. Results of the National Assessment of Education Progress (NAEP) and the Third International Math and Science Survey (TIMSS) suggest that U.S. students are competent in basic computation but lack the understanding to solve unfamiliar problems (Stevenson 1998; Wilson and Blank 1999).

Ironically, the NAEP and TIMSS studies also suggest that the teaching methods that make students successful in basic computation also hinder understanding. The most common method for teaching children computation is teaching the standard procedures (algorithms, such as regrouping), then providing enough practice that children can do the procedures on their own (Stigler and Hiebert 1997). This type of instruction has been called “parrot math” (O’Brien 1999) because children do not have to think about the meaning of numbers at all; they simply memorize and repeat the teacher’s directions.

Because standard algorithms have often been criticized as detrimental to understanding, educators have developed more child-friendly procedures. These alternative procedures more closely align with children’s natural ways of solving problems (Randolph and Sherman 2001). For example, they have children add two-digit numbers by first adding the tens, the way most children naturally add. But are these alternative procedures really much better than standard procedures? Carpenter et al. (1998) have suggested that the explicit teaching of alternative procedures may be only slightly better than teaching children to regroup; children

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**How Computation Is Taught**

By Dan Heuser
still are parroting the teacher’s directions instead of thinking about the meaning of numbers.

A third approach has been to encourage children to invent their own procedures. These invented procedures are the result of a child being given problems but deliberately not being told how to solve them. With proper support, the child can work from his or her own knowledge to fashion a solution strategy that is highly personalized and rooted in understanding. For example, a child with a strong grasp of tens and ones place value would likely create computational procedures that draw on that knowledge, whereas a child with less developed ten-knowledge would fashion strategies that do not rely as heavily on place value. Why have children invent ways of computing when they can learn either standard or alternative procedures? Because invented procedures stem from children’s existing understanding, students are more likely to construct knowledge that is personal and robust, rather than memorize what they may not understand. Indeed, understanding of number and operations
has proved to be a strength among children who invent their own procedures (Carpenter et al. 1998; Kamii and Dominick 1997, 1998).

In my district, we use a mathematics program called Everyday Mathematics (Everyday Learning Corporation 1995). The program begins by encouraging invention. After nine lessons, it presents alternative procedures. This approach concerns me because too few children invent their own strategies, and children often abandon whatever procedures they invented as soon as they are taught alternative strategies. This observation led to my third goal: allowing the students to invent their own ways of being fluent in computation, without exposure to any procedure, either standard or alternative. My role moved from giving children procedures to memorize to supporting students as they create and refine their own strategies.

Project Background

Other researchers have developed the use of the candy context to teach computation (McClain, Cobb, and Bowers 1998; Whitenack et al. 2001). To encourage student invention, this project enhanced the candy scenario in two ways. First, each lesson was conducted using the workshop format (Heuser 2000a, 2000b, 2002). Workshops are designed to teach mathematics and science through the systematic use of four instructional elements that extensive research supports: hands-on instruction, reflection, problem solving, and choice (see Heuser 2002 for a review). Second, the instruction focused on helping children understand the triangular relationship between physical quantity, number symbol, and number name (Fuson, Smith, and Lo Cicero 1997). In the past, I have found that children’s computation suffers because they often do not see the connection between different representations of number.

I teach mathematics for two multiage first- and second-grade classrooms. First graders are grouped together for mathematics, as are second graders. The lesson sequence was composed of thirty, one-hour lessons, divided into three parts: introduction, addition, and subtraction.

Introducing the Lyon Candy Factory through Hands-on Activity

To introduce the candy-factory context, I told students that they would be working in the packing room of the Lyon Candy Company. After discussing how Tootsie Rolls are packed, I demonstrated the simulation and explained, “Our Unifix cube candies are wrapped in similar ways. Most of our candies are packed into groups of ten candies that are stuck together, like this. These are called rolls. Any candies left over we sell individually. Each is called a piece.”

The students’ job for the next few days was to work with a partner, packing candies and reporting how many they packed. I warned the children about their prickly boss, Mr. Punchbowl: “He is

Figure 1

Questions the teacher asked during the activity

- How many candies do you have altogether? How many rolls? How many pieces?
- You say that you packed 67 candies altogether. I’ll write “67” on this paper. Now, what does this part [pointing to the 6] of 67 mean? Show me with your candies. What does this part [indicating the 7] mean?
- Show me with the candies where the “sixty” part of 67 is. Where is the “seven” part?
- How many more candies would you need to have 70? . . . 100? How many less to have 60? . . . 50?
very particular about how the work should be done. All rolls should have ten candies—no more, no less. Mr. Punchbowl wants to know exactly how many you packed, and he wants to know quickly. Follow these rules and you will stay on Mr. Punchbowl’s good side.”

Students packed the candy over the next three days. Hands-on activities are very motivating and help illustrate a number of important relationships. As Mr. Punchbowl’s intermediary, I conferenced with each group, asking them how many candies the students had packed, as well as some of the other questions shown in figure 1. By helping children reflect on the relationships between physical quantity (three rolls and five pieces), symbols (35), and number words (thirty-five), I hoped to lay a firm foundation for students’ inventions of efficient tens-based procedures.

Each workshop ended with students responding to a reflective prompt. First, the pairs recorded how much they packed, using a shorthand negotiated by Mr. Punchbowl (through me) and the class (see fig. 2). Second, we discussed different problems related to two imaginary workers named Katie and Benji. For example, I asked, “If Katie packed four rolls and three pieces, how many pieces of candy did she pack altogether? Explain how you know.”

### Addition

The fourth lesson began the problem-solving format that would continue for much of the remainder of the unit. First, a story problem based on the candy-factory context was presented on the overhead. Initially, I showed quantity in the candy shorthand (see fig. 3) to help children make the connection between the numbers and the drawings. Throughout the unit, however, written numbers gradually replaced the shorthand.

The students solved the problem individually at their desks in any way that they could. As the children worked, I walked around and took note of the different strategies that they used. Two or three children (selected so that a variety of methods were represented) were asked to solve the problem one at a time in front of their classmates.

At first, most students copied the symbols from the overhead and then counted to find the total. Soon, however, some children began to invent written or mental procedures. Jenny, for example, used the method shown in figure 4 to solve 56 + 35. Other students adapted and refined this method, and other related procedures, over the course of the unit.

Each mathematics workshop follows the format of mini-lesson, activity period, and reflection. During the mini-lessons, we repeated the process, presenting problems for the children to solve and share their strategies. The activity period often consisted of different games designed to support place value, fact recall, and operations.

These games were useful in two ways: They helped children learn the basic mathematics facts and they aided children’s computational fluency. Each workshop concluded with children replying to a reflective prompt. Responses were verbal, written, or both.
Reflection and Choice

Aside from problem solving and hands-on activity, reflection and choice are two essential elements in mathematics workshops. In this sequence, reflection often took the form of students sharing and listening to other students’ strategies. The students were told that (1) several different ways to solve problems always exist; (2) each child’s responsibility is to explain his or her procedure as well as to try to make sense of the procedures of others; and (3) this reflection is an important part of deepening the students’ understanding.

Reflection was especially useful to encourage children to move from using drawings to directly model problems (such as in figure 5a) to using purely numerical procedures. Children who normally made drawings were often paired with children who used numbers, and then asked to tell each other and write about how they solved the same problem (see fig. 6). This gave drawers another opportunity to see a more sophisticated strategy and also helped students who used numbers but typically did not record their work. Researchers have found that the latter group of students often has difficulty with more complex problems because they have not acquired recording skills (Fuson et al. 1997).

As partners reflected, I sat in on some of their conversations and helped them articulate their ideas or offered suggestions. For example, Jane was listening as Lisa described her new numerical procedure for solving $29 + 52$ (see fig. 6). Jane primarily used the candy drawings to add, but because I believed she was ready to incorporate some of Lisa’s method, I pointed out a similarity between the two methods: “You draw the tens first and then add them. That’s just like how Lisa does it, only she just adds the numbers of tens without doing the...

**Figure 4**

Jenny’s method

\[
\begin{align*}
56 + 35 &= 91 \\
50 + 30 &= 80 \\
6 + 5 &= 11 \\
11 + 80 &= 91
\end{align*}
\]
drawing. I bet you can do that too. It might save you some time to not have to do the drawings. Let me give you another problem, and you show me how you do it!"

Mathematics workshops also allow students the choice of how to solve problems and encourage children to choose procedures that make sense to them. This project did not require students to use a given procedure; however, I made it clear that the responsibility to find a solution method lay with each of them. When children said, “I don’t know how to solve this!” I tried to guide them to connect with something that they did understand. Simplifying the problem slightly usually was enough to lead them toward a solution. For example, I might say, “You’re having trouble with 45 + 27? Well, how much is 45 + 20? Now how can you figure out 45 + 27?” Another approach was to ask if a drawing would help them.

**Subtraction**

Multidigit subtraction was a goal only for the second graders. We used the same lesson format of problem solving, game playing, and reflecting. At first, most subtraction problems involved sending out candies that had been packed. As in addition, we eventually moved away from candy-factory stories to other word problems as well as straightforward equations. Figures 5 and 7 show examples of correct and incorrect student strategies.

Children have far more difficulty with subtraction than with addition. One problem during the first year of the project stemmed from my students’ unfamiliarity with negative numbers. Some students concluded that taking a greater number away from a lesser one (such as the ones digits in 52 – 37) equaled zero. Another difficulty was that some students had not yet mastered the basic subtraction facts. For these children, figuring out both the subtraction facts and the larger problem often was too demanding a task. Many also seemed to struggle with the part-whole relationship. During the second year, I delayed teaching computation by a month to address these issues. We spent that time reinforcing subtraction facts and introducing negative numbers through such games as Below Zero.

Below Zero uses two dice and a board shaped like a thermometer and numbered from 5º at the top to –5º at the bottom. Players take turns rolling the dice and subtracting the two numbers to find the difference. A six and a two, for example, can be either 6 – 2 or 2 – 6. The player then covers the
appropriate spot on his or her game board (4º or -4º in this case). The winner is the first to cover his or her entire board.

I provided more help in subtraction by tutoring small groups of students—grouping those who used similar strategies—for a few minutes while the other students played games. The aim of the sessions was to work the bugs out of children’s strategies. For example, Brian had adopted the subtraction strategy shown in figure 5a. He often began by subtracting the numbers in the tens place. In this example, the result was four tens. Then he overgeneralized, however, by misrepresenting 3 – 5 as negative two tens, not as negative two ones. I suggested that Brian write out his calculations in a bit more detail, as shown in figure 5b, then gave him several similar problems to practice this refined procedure. These children made rapid progress and most became adept at subtraction after five tutoring periods.

Fuson et al. (1997) provided two other subtraction suggestions, and they seemed to help when I tried them the second year. First, the authors found that students tend to overgeneralize strategies when addition and subtraction are taught separately. The authors suggested that teachers intermix addition and subtraction problems fairly early in the unit. Second, they found that mixing three-digit computation with two-digit computation helped children see place-value patterns more easily than when all the focus was on two-digit numbers. Three-digit computation fit in nicely with the candy-factory context because a bag contains ten rolls, or one hundred pieces.

Results

How successful were the children in reaching the project goals? In straightforward computation, all students could correctly solve most problems. Their rate of success was as least as good as that of my students in previous years, whom I directly taught alternative procedures. Much of this achievement seemed to come from students’ facility with a variety of procedures, which allowed them to flexibly choose the procedure most appropriate for the problem.

Although I suspected that understanding would be a strong point of these students, the degree of their comprehension was surprising. Students’ understanding of number and operations was far better than that of children taught traditionally, based on their performance on a number of com-
plex or nonroutine tasks. On two questions taken from the NAEP, for example, the first graders matched the performance of fourth graders, whereas the second graders nearly doubled the older students’ rate of success.

A goal of this project was to encourage children’s inventions of numerical computational strategies—not just drawings used to directly model problems. About half of the first graders and three-quarters of the second graders were using invented strategies for addition at the end of the unit. Invented strategies for subtraction were less common; only half of the second graders used them. Although some children were reluctant to move beyond the drawings, the drawings were an effective and understandable way to solve problems for many of them. Students became so adept at drawing that their fluency often matched that of students using written number inventions.

Is There a Place for Standard and Alternative Procedures?

Given the outcome of this project, it can be argued that the children did not suffer by not being taught computation procedures, either standard or alternative. The children seemed comfortable with their various methods. Introducing procedures that might supplant their thinking did not make sense. Moreover, the understanding that students gained from inventing likely would be shortchanged if the focus moved away from invention.

Still, I believe that mathematics class has a place for alternative (and even standard) procedures. A fine line exists between having a child parrot a solution method and thoughtfully introducing to a child (or a small group of children) a procedure that is a natural extension of their inventions. For example, the invented procedure in figure 8 is just a step away from the standard algorithm. A conversation between that child and the teacher might sound like the following:

The way that you do it is very similar to this way. You do the tens first, and then add another ten if there are ten or more. This other way the ones are added first, and then carry the ten over into the tens place. Then you add the tens together. It’s really the same as yours, except with this one you don’t have to cross out the tens. Do you want to try this new way on a new problem with me now, or are you happy with how you do it?

References